Aspects of braneworld cosmology and holography*

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Abstract

In a holographic braneworld universe a cosmological fluid occupies a 3+1 dimensional brane located at the boundary of the asymptotic AdS_5 bulk. The AdS/CFT correspondence and the second Randall-Sundrum model are combined to establish a relationship between the RSII braneworld cosmology and the boundary metric induced by the time dependent bulk geometry. Some physically interesting scenarios are discussed in the framework of the Friedmann Robertson Walker cosmology involving the RSII and holographic braneworlds.

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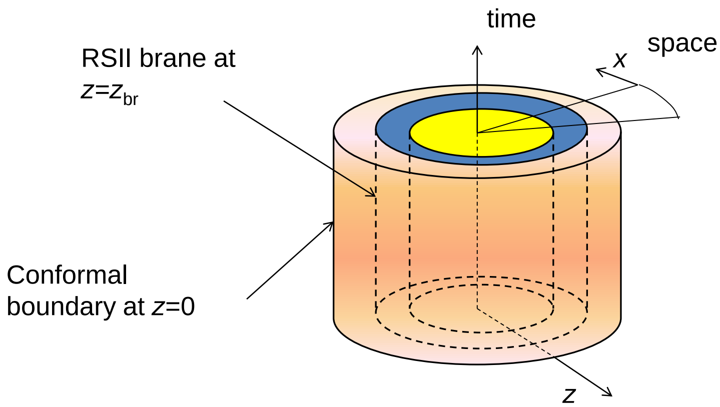

Figure 1: Illustration of the AdS_5 bulk with two branes: RSII brane located at $z = z_{\rm br}$ and the holographic brane at $z = 0$.

1. Introduction

Branewarld cosmology is based on the scenario in which matter is confined on a brane moving in the higher dimensional bulk with only gravity allowed to propagate in the bulk [1, 2, 3, 4]. The brane can be placed, e.g., at the boundary of a 5-dim asymptotically Anti de Sitter space (AdS_5). Anti de Sitter space is dual to a conformal field theory at its boundary through the so called AdS/CFT correspondence [5]. This correspondence reflects an obvious symmetry relationship: On the one hand, AdS_5 is a maximally symmetric solution to Einsteins equations with negative cosmological constant with the symmetry group $AdS_5 \equiv$ SO(4,2). On the other hand, the 3+1 boundary conformal field theory is invariant under conformal transformations: Poincaré + dilatations + special conformal transformation. These transformations constitute the conformal group $\equiv$ SO(4,2).

We will consider two types of braneworlds (Fig. 1): 1) Holographic braneworld in with a 3-brane located at the boundary of the asymptotic AdS_5. The cosmology is governed by matter on the brane in addition to the boundary CFT. 2) Randall-Sundrum braneworld with a 3-brane located at a finite distance from the boundary of AdS_5. We will demonstrate that there exists a map between these two substantially different scenarios. Most of the material presented here is based on [6] and earlier works [7, 8, 9, 10].

We use the metric signature $(+,- - --)$ and curvature convention

$R^a{}_{bcd} = \partial_c \Gamma^a_{db} - \partial_d \Gamma^a_{cb} + \Gamma^e_{db}\Gamma^a_{ce} - \Gamma^e_{cb}\Gamma^a_{de}$ and $R_{ab} = R^s{}_{asb}$, so that Einstein's equations are $R_{ab} - \frac{1}{2}RG_{ab} = +8\pi G T_{ab}$.

2. Randall-Sundrum model

2.1. Basics

The Randall-Sundrum (RS) model [1, 2]. is a simple physically relevant model related to AdS/CFT. The model was originally proposed as a solution to the hierarchy problem in particle physics and as a possible mechanism for localizing gravity on the 3+1 dimensional universe embedded in a 4+1 spacetime without compactification of the extra dimension. It was soon realized that the RS model is deeply rooted in a wider framework of AdS/CFT correspondence [11, 12, 13, 14, 15, 16, 17].

The Randall-Sundrum model is a 4+1-dimensional universe with AdS_5 geometry containing two 3-branes with opposite brane tensions separated in the 5th dimension. The total action is a sum

$$S = S_{\text{bulk}} + S_{\text{GH}} + S_{\text{br1}} + S_{\text{br2}}, \tag{1}$$

where

$$S_{\text{bulk}} = \frac{1}{8\pi G_5} \int d^5x \sqrt{G} \left[-\frac{R^{(5)}}{2} - \Lambda_5 \right], \tag{2}$$

is the bulk action, Λ_5 being the bulk cosmological constant related to the AdS curvature radius as $\Lambda_5 = -6/\ell^2$. The remaining terms are the Gibbons-Hawking boundary term

$$S_{\text{GH}}[h] = \frac{1}{8\pi G_5} \int_\Sigma d^4x \sqrt{-h} K[h]. \tag{3}$$

and two brane actions of the form

$$S_{\text{br}}[h] = \int_\Sigma d^4x \sqrt{-h}(-\sigma + \mathcal{L}^{\text{matt}}[h]). \tag{4}$$

Here we denote by G the determinant of the bulk metric $G_{\mu\nu}$, by h the determinant of the metric $h_{\mu\nu}$ induced on the hypersurface Σ, and by σ the brane tension. Matter on the brane is described by the Lagrangian $\mathcal{L}^{\text{matt}}$.

In the following we will make use of various coordinate systems:

1. Fefferman-Graham coordinates

$$ds^2_{(5)} = G_{ab}dx^a dx^b = \frac{\ell^2}{z^2}\left(g_{\mu\nu}dx^\mu dx^\nu - dz^2\right), \tag{5}$$

2. Gaussian normal coordinates

$$ds^2_{(5)} = e^{-2y/\ell} g_{\mu\nu}dx^\mu dx^\nu - dy^2, \tag{6}$$

3. Schwarzschild coordinates

$$ds^2_{\rm ASch} = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2 d\Omega_\kappa^2, \tag{7}$$

where

$$f(r) = \frac{r^2}{\ell^2} + \kappa - \mu\frac{\ell^2}{r^2}, \tag{8}$$

and

$$d\Omega_\kappa^2 = d\chi^2 + \frac{\sin^2(\sqrt{\kappa}\chi)}{\kappa}(d\vartheta^2 + \sin^2\vartheta d\varphi^2) \tag{9}$$

is the spatial line element for a closed ($\kappa = 1$), open hyperbolic ($\kappa = -1$), or open flat ($\kappa = 0$) space. The dimensionless parameter μ is related to the black-hole mass via [18, 19]

$$\mu = \frac{8G_5 M_{\rm bh}}{3\pi\ell^2}. \tag{10}$$

These coordinate representations are related via simple coordinate transformations

$$z = e^{y/\ell}, \quad \frac{r^2}{\ell^2} = \frac{\ell^2}{z^2} - \frac{\kappa}{2} + \frac{\kappa^2 + 4\mu}{16}\frac{z^2}{\ell^2}. \tag{11}$$

2.2. Second Randall-Sundrum model (RSII)

The RSII model [2] was proposed as an alternative to compactification of extra dimensions. A compactification of extra dimensions is necessary to localize gravity on the 3+1 dimensional universe. If extra dimensions were large that would yield unobserved modification of Newton's gravitational law. Experimental bound on the volume of n extra dimensions is [20]

$$V^{1/n} \leq 0.1\text{mm}. \tag{12}$$

RSII brane-world does not rely on compactification to localize gravity at the brane, but on the curvature of the bulk ("warped compactification). The negative cosmological constant Λ_5 acts to squeeze the gravitational field closer to the brane. One can see this in Gaussian normal coordinates (6) with an exponentially attenuating warp factor $e^{-2\ell y}$.

In RSII observers reside on the positive tension brane at $y = 0$ and the negative tension brane is pushed off to infinity in the fifth dimension. In the original RSII model one assumes the Z_2 symmetry $z \leftrightarrow z_{\rm br}^2/z$, so the region $0 < z \leq z_{\rm br}$ is identified with $z_{\rm br} \leq z < \infty$, with the observer brane at the fixed point $z = z_{\rm br}$. Hence, the braneworld is sitting between two patches of AdS$_5$, one on either side, and is therefore dubbed "two-sided" [15, 17]. In contrast, in the "one-sided" RSII model the region $0 \leq z \leq z_{\rm br}$ is simply cut off so the bulk is the section of spacetime $z_{\rm br} \leq z < \infty$.

The Planck mass scale is determined by the curvature of the five-dimensional space-time

$$\frac{1}{G_{\rm N}} = \frac{\gamma}{G_5}\int_{y_{\rm br}}^{\infty} dy\psi^2 = \frac{\gamma\ell}{2G_5}. \tag{13}$$

where we have introduced the "sidedness" parameter γ [6] to facilitate a joint description of the two versions of RSII model: the one-sided ($\gamma = 1$) and two-sided ($\gamma = 2$). One usually imposes a fine tuning condition on the brane tension

$$\sigma = \sigma_0 \equiv \frac{3\gamma}{8\pi G_5 \ell} = \frac{3}{4\pi G_{\rm N}\ell^2}. \tag{14}$$

which eliminates the 4-dim cosmological constant. Note that the RSII fine tuning condition does not depend on the sidedness γ if σ_0 is expressed in terms of the four-dimensional Newton constant.

Table top measurements of the Newton gravitational law impose a bound on the AdS_5 curvature radius. The classical 3+1 dimensional gravity is altered on the RSII brane due to the extra dimension. For $r \gg \ell$ the weak gravitational potential created by an isolated matter source on the brane is given by [21]

$$\Phi(r) = \frac{G_{\rm N}M}{r}\left(1 + \frac{2\ell^2}{3r^2}\right). \tag{15}$$

Table-top tests of Long et al [20] find no deviations of Newton's potential at distances greater than 0.1 mm and place the limit curvature

$$\ell < 0.1\text{mm}, \quad \text{or} \quad \ell^{-1} > 10^{-12}\text{GeV}. \tag{16}$$

2.3. RSII cosmology – Dynamical brane

Branewarld cosmology is based on the scenario in which matter is confined on a brane moving in the higher dimensional bulk with only gravity allowed to propagate in the bulk [1, 2, 3, 4]. In this section we give a simple derivation of the RSII braneworld cosmology following J. Soda [22]. Cosmology on the brane is obtained by allowing the brane to move in the bulk. Equivalently, one could keep the brane fixed at $y = 0$ while making the metric in the bulk time dependent.

Consider a time dependent brane hypersurface defined by

$$r - a(t) = 0, \tag{17}$$

in AdS-Schwarzschild background [23, 24] where $a = a(t)$ is an arbitrary positive function. The induced line element on the brane is

$$ds^2_{\rm ind} = n^2(t)dt^2 - a(t)^2 d\Omega^2_\kappa, \tag{18}$$

where

$$n^2 = f(a) - \frac{(\partial_t a)^2}{f(a)}, \tag{19}$$

and f is defined by (8). The junction conditions on the brane with matter

$$K_{\mu\nu}|_{r=a-\epsilon} = \frac{8\pi G_5}{3\gamma}(\sigma g_{\mu\nu} + 3T_{\mu\nu}) \tag{20}$$

yield

$$\frac{(\partial_t a)^2}{n^2 a^2} + \frac{f}{a^2} = \frac{1}{\ell^2 \sigma_0^2}(\sigma + \rho)^2. \tag{21}$$

Now, imposing the fine tuning condition (14) one finds a modified Friedmann equation [25, 26, 27, 28].

$$\mathcal{H}^2 = \frac{8\pi G_\mathrm{N}}{3}\rho + \left(\frac{4\pi G_\mathrm{N}\ell}{3}\right)^2 \rho^2 + \frac{\mu\ell^2}{a^4}, \tag{22}$$

where

$$\mathcal{H}^2 = H^2 + \frac{\kappa}{a^2}, \quad H = \frac{\partial_t a}{na}. \tag{23}$$

Equation (22) differs from the standard Friedmann equation by the last two terms on the right-hand side. RSII cosmology is thus subject to astrophysical and cosmological tests (see, e.g., Refs. [29, 30]). The deviation proportional to ρ^2 poses no problem as it decays as a^{-8} in the radiation epoch and will rapidly become negligible after the end of the high-energy regime $\rho \simeq \sigma_0$. The last term on the right-hand side, the so called "dark radiation", for positive μ should not exceed 10% of the total radiation content in the epoch of BB nucleosynthesis whereas for negative μ could be as large as the rest of the radiation content [31, 32]. As expected, both one-sided and two-sided versions of the RSII model yield identical braneworld cosmologies.

The second Friedmann equation may be easily obtained by combining the time derivative of (22) with the energy conservation equation

$$\partial_t \rho + 3(\rho + p)\frac{\partial_t a}{a} = 0. \tag{24}$$

3. Connection with AdS/CFT

AdS/CFT correspondence is a holographic duality between gravity in d+1-dimensional space-time and quantum conformal field theory (CFT) on the d-dim boundary. Original formulation stems from string theory: the original AdS/CFT conjecture establishes an equivalence of a four dimensional $\mathcal{N} = 4$ supersymmetric Yang-Mills theory and string theory in a ten dimensional $\mathrm{AdS}_5 \times \mathrm{S}_5$ bulk [5, 33, 34].

3.1. RSII braneworld as a cutoff in AdS_5

In the RSII model by introducing the boundary in AdS_5 at $z = z_{br}$ instead of $z = 0$, the model is conjectured to be dual to a cutoff CFT coupled to gravity, with $z = z_{br}$ providing the IR cutoff (corresponding to the UV cutoff of the boundary CFT) [15]. In the one-sided RSII model, the model involves a single CFT at the boundary of a single patch of AdS_5. In the two-sided RSII model one would instead require two copies of the CFT, one for each of the AdS_5 patches.

The on-shell bulk action

$$S_{\text{bulk}} = \frac{1}{8\pi G_5}\int d^5x\sqrt{G}\left[-\frac{R^{(5)}}{2} - \Lambda_5\right], \tag{25}$$

is infrared divergent because physical distances diverge at $z = 0$. The asymptotically AdS metric near $z = 0$ can be expanded as

$$ds^2_{(5)} = \frac{\ell^2}{z^2}\left(g_{\mu\nu}dx^\mu dx^\nu - dz^2\right), \tag{26}$$

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + z^6 g^{(6)}_{\mu\nu} + \dots \,. \tag{27}$$

Explicit expressions for $g^{(2n)}_{\mu\nu}$, $n = 1, 2, 3$ in terms of arbitrary $g^{(0)}_{\mu\nu}$ may be found in Ref. [35]. In particular, we will need

$$g^{(2)}_{\mu\nu} = \frac{1}{2}\left(R_{\mu\nu} - \frac{1}{6}Rg^{(0)}_{\mu\nu}\right) \tag{28}$$

and the relation

$$\text{Tr}g^{(4)} = -\frac{1}{4}\text{Tr}(g^{(2)})^2, \tag{29}$$

where the trace of a tensor $A_{\mu\nu}$ is defined as

$$\text{Tr}A = A^\mu_\mu = g^{(0)\mu\nu}A_{\mu\nu}. \tag{30}$$

Now, we regularize the action by placing the RSII brane near the AdS boundary, i.e., at $z = \epsilon\ell$, $\epsilon \ll 1$, so that the induced metric is

$$h_{\mu\nu} = \frac{1}{\epsilon^2}(g^{(0)}_{\mu\nu} + \epsilon^2\ell^2 g^{(2)}_{\mu\nu} + \dots). \tag{31}$$

The bulk splits in two regions: $0 \leq z < \epsilon\ell$, and $\epsilon\ell \leq z \leq \infty$. We can either discard the region $0 \leq z < \epsilon\ell$ (one-sided regularization, $\gamma = 1$) or invoke the Z_2 symmetry and identify two regions (two-sided regularization, $\gamma = 2$). Then, the regularized bulk action is

$$S^{\text{reg}}_{\text{bulk}} = \gamma S^{\text{reg}}_0 = \frac{\gamma}{8\pi G_5}\int_{z\geq\epsilon\ell} d^5x\sqrt{G}\left[-\frac{R^{(5)}}{2} - \Lambda_{(5)}\right] + S_{\text{GH}}[h] \tag{32}$$

The renormalized action is obtained by adding counterterms to $S_0^{\rm reg}$ and taking the limit $\epsilon \to 0$ [35, 36]

$$S_0^{\rm ren}[g^{(0)}] = \lim_{\epsilon\to 0}(S_0^{\rm reg}[G] + S_1[h] + S_2[h] + S_3[h]), \tag{33}$$

The necessary counterterms are [35]

$$S_1[h] = -\frac{6}{16\pi G_5 \ell}\int d^4x\sqrt{-h}, \tag{34}$$

$$S_2[h] = -\frac{\ell}{16\pi G_5}\int d^4x\sqrt{-h}\left(-\frac{R[h]}{2}\right), \tag{35}$$

$$S_3[h] = -\frac{\ell^3}{16\pi G_5}\int d^4x\sqrt{-h}\frac{\log\epsilon}{4}\left(R^{\mu\nu}[h]R_{\mu\nu}[h] - \frac{1}{3}R^2[h]\right). \tag{36}$$

Now we demand that the variation with respect to the induced metric $h_{\mu\nu}$ of the total RSII action (the sum of the regularized on shell bulk action and the brane action (4)) vanishes, i.e., we require

$$\delta(S_{\rm bulk}^{\rm reg}[h] + S_{\rm br}[h]) = 0, \tag{37}$$

which may be expressed as

$$\delta\left[\gamma S_0^{\rm ren} - \gamma S_3 - \left(\sigma - \frac{3\gamma}{8\pi\ell G_5}\right)\int d^4x\sqrt{-h} + \int d^4x\sqrt{-h}\mathcal{L}_{\rm matt} \right. \\ \left. + \frac{\gamma\ell}{16\pi G_5}\int d^4x\sqrt{-h}\frac{R[h]}{2}\right] = 0. \tag{38}$$

The third term gives the contribution to the cosmological constant and may be eliminated by imposing the RSII fine tuning condition (14). The variation of the scheme dependent S_3 may be combined with the first term. Then, according to the AdS/CFT prescription by functionally differentiating the renormalized on-shell bulk gravitational action with respect to the boundary metric $g_{\mu\nu}^{(0)}$ one obtains the expectation value $\langle T_{\mu\nu}^{\rm CFT}\rangle$ and hence

$$\delta(S_0^{\rm ren} - S_3) = \frac{1}{2}\int d^4x\sqrt{-h}\langle T_{\mu\nu}^{\rm CFT}\rangle\delta h^{\mu\nu}, \tag{39}$$

With this the variation of the action yields Einsteins equations on the boundary

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G_{\rm N}(\gamma\langle T_{\mu\nu}^{\rm CFT}\rangle + T_{\mu\nu}^{\rm matt}), \tag{40}$$

where the energy-momentum tensor $T^{\rm matt}_{\mu\nu}$ describes matter on the holographic brane in addition to the holographic conformal part given by [35]

$$\langle T^{\rm CFT}_{\mu\nu}\rangle = -\frac{\ell^3}{4\pi G_5}\left\{g^{(4)}_{\mu\nu} - \frac{1}{8}\left[({\rm Tr}g^{(2)})^2 - {\rm Tr}(g^{(2)})^2\right]g^{(0)}_{\mu\nu} \right. \\ \left. -\frac{1}{2}(g^{(2)})^2_{\mu\nu} + \frac{1}{4}{\rm Tr}g^{(2)}g^{(2)}_{\mu\nu}\right\}. \qquad (41)$$

This is an explicit realization of the AdS/CFT correspondence: the vacuum expectation value of a boundary CFT operator is obtained solely in terms of geometrical quantities of the bulk.

3.2. Conformal anomaly

It is of particular interest to check whether the stress tensor $T^{\rm CFT}$ obtained using AdS/CFT prescription correctly reproduces the conformal anomaly. From (41) with the help of (28) and (29) we find

$$\langle T^{{\rm CFT}\mu}{}_{\mu}\rangle = \frac{\ell^3}{128\pi G_5}\left(G_{\rm GB} - C^2\right), \qquad (42)$$

where

$$G_{\rm GB} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2 \qquad (43)$$

is the Gauss-Bonnet invariant and

$$C^2 \equiv C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 2R^{\mu\nu}R_{\mu\nu} + \frac{1}{3}R^2 \qquad (44)$$

is the square of the Weyl tensor $C_{\mu\nu\rho\sigma}$. This result should be compared with the standard conformal anomaly calculated in field theory [37]

$$\langle T^{{\rm CFT}\mu}{}_{\mu}\rangle = bG_{\rm GB} - cC^2 + b'\Box R. \qquad (45)$$

The two results agree if we ignore the last term in (45) and identify

$$b = c = \frac{G_5}{128\pi\ell^3}. \qquad (46)$$

The standard field theory calculations give [37, 38]

$$b = \frac{n_{\rm s} + (11/2)n_{\rm f} + 62n_{\rm v}}{360(4\pi)^2}, \quad c = \frac{n_{\rm s} + 3n_{\rm f} + 12n_{\rm v}}{120(4\pi)^2}. \qquad (47)$$

where $n_{\rm s}$, $n_{\rm f}$, $n_{\rm v}$ are the numbers of massless scalar bosons, Weyl fermions and vector bosons, respectively. Hence, generally $b \neq c$. However, in the $\mathcal{N} = 4$ U(N) super-Yang-Mills theory, $n_{\rm s} = 6N^2$, $n_{\rm f} = 4N^2$, and $n_{\rm v} = N^2$, in which case the equality $b = c$ holds and the conformal anomaly is correctly reproduced by the holographic expression (42) if we identify [39]

$$\frac{\ell^3}{G_5} = \frac{2N^2}{\pi}. \qquad (48)$$

4. Holographic cosmology

As we have shown in the previous section, in the limit in which the RSII brane approaches the AdS_5 boundary, the geometry on the boundary brane, referred to as the *holographic brane*, satisfies a special form of Einstein's equations (40). To derive the corresponding cosmology we start from AdS-Schwarzschild static coordinates in the bulk and make the coordinate transformation

$$t = t(\tau, z), \quad r = r(\tau, z). \tag{49}$$

The line element will take a general form

$$ds^2_{(5)} = \frac{\ell^2}{z^2}\left(n^2(\tau, z)d\tau^2 - a^2(\tau, z)d\Omega^2_\kappa - dz^2\right), \tag{50}$$

Imposing the boundary conditions at $z = 0$:

$$n(\tau, 0) = 1, \quad a(\tau, 0) = a_{\rm h}(\tau), \tag{51}$$

we obtain the induced metric at the boundary in the general FRW form

$$ds^2_{(0)} = g^{(0)}_{\mu\nu}dx^\mu dx^\nu = d\tau^2 - a^2_{\rm h}(\tau)d\Omega^2_\kappa. \tag{52}$$

Solving Einsteins equations in the bulk one finds [8]

$$a^2 = a^2_{\rm h}\left[\left(1 - \frac{\mathcal{H}^2_{\rm h}z^2}{4}\right)^2 + \frac{1}{4}\frac{\mu z^4}{a^4_{\rm h}}\right], \quad \mathcal{N} = \frac{\dot{a}}{\dot{a}_{\rm h}}. \tag{53}$$

where

$$\mathcal{H}^2_{\rm h} = H^2_{\rm h} + \frac{\kappa}{a^2_{\rm h}}, \tag{54}$$

and $H_{\rm h} = \dot{a}_{\rm h}/a_{\rm h}$ is the Hubble expansion rate on boundary. Comparing the exact $g_{\mu\nu}(\tau, z)$ in (50) with the expansion (26) we can extract $g^{(2)}_{\mu\nu}$ and $g^{(4)}_{\mu\nu}$. Then, using the expression (41) we obtain

$$\langle T^{\rm CFT}_{\mu\nu}\rangle = t_{\mu\nu} + \frac{1}{4}\langle T^{{\rm CFT}\,\alpha}{}_{\alpha}\rangle g^{(0)}_{\mu\nu}, \tag{55}$$

where the second term on the right-hand side corresponds to the conformal anomaly

$$\langle T^{{\rm CFT}\,\alpha}{}_{\alpha}\rangle = \frac{3\ell^3}{16\pi G_5}\frac{\ddot{a}_{\rm h}}{a_{\rm h}}\mathcal{H}^2_{\rm h}, \tag{56}$$

and the first term is a traceless tensor with non-zero components

$$t_{00} = -3t^i_i = \frac{3\ell^3}{64\pi G_5}\left(\mathcal{H}^4_{\rm h} + \frac{4\mu}{a^4_{\rm h}} - \frac{\ddot{a}_{\rm h}}{\dot{a}_{\rm h}}\mathcal{H}^2_{\rm h}\right). \tag{57}$$

Hence, apart from the conformal anomaly, the CFT dual to the time dependent asymptotically AdS_5 bulk metric is a conformal fluid with the equation of state $p_{\mathrm{CFT}} = \rho_{\mathrm{CFT}}/3$, where $\rho_{\mathrm{CFT}} = t_{00}$, $p_{\mathrm{CFT}} = -t^i_i$.

Using this, from the boundary Einstein equations we obtain the holographic Friedmann equation [8, 7]

$$\mathcal{H}_{\mathrm{h}}^2 = \frac{\ell^2}{4}\left(\mathcal{H}_{\mathrm{h}}^4 + \frac{4\mu}{a_{\mathrm{h}}^4}\right) + \frac{8\pi G_{\mathrm{N}}}{3}\rho_{\mathrm{h}}. \tag{58}$$

Here we have used the energy-momentum tensor with nonvanishing components

$$T_{00}^{\mathrm{matt}} = \rho_{\mathrm{h}}, \quad T_{ij}^{\mathrm{matt}} = p_{\mathrm{h}} g_{ij}^{(0)}, \tag{59}$$

where ρ_{h} and p_{h} are the matter energy density and pressure, respectively. The second Friedmann equation can be derived by combining the time derivative of (58) with the energy conservation equation

$$\dot{\rho}_{\mathrm{h}} + 3(\rho_{\mathrm{h}} + p_{\mathrm{h}})H_{\mathrm{h}} = 0. \tag{60}$$

One finds

$$\frac{\ddot{a}_{\mathrm{h}}}{a_{\mathrm{h}}}\left(1 - \frac{\ell^2}{2}\mathcal{H}_{\mathrm{h}}^2\right) + \mathcal{H}_{\mathrm{h}}^2 = \frac{4\pi G_{\mathrm{N}}}{3}(\rho_{\mathrm{h}} - 3p_{\mathrm{h}}). \tag{61}$$

5. Holographic map

The time dependent bulk spacetime with metric (50) may be regarded as a z-foliation of the bulk with FRW cosmology on each z-slice [6]. In particular, at $z = z_{\mathrm{br}}$ one has the RSII cosmology and at at $z = 0$ the holographic cosmology. A map between a z-cosmology and $z = 0$-cosmology can be constructed using (53) and the inverse relation

$$a_{\mathrm{h}}^2 = \frac{a^2}{2}\left(1 + \frac{\mathcal{H}^2 z^2}{2} + \mathcal{E}\sqrt{1 + \mathcal{H}^2 z^2 - \frac{\mu z^4}{a^4}}\right), \tag{62}$$

where

$$\mathcal{E} = \begin{cases} -1, & \text{for two-sided version,} \\ \pm 1, & \text{for one-sided version.} \end{cases} \tag{63}$$

A functional relationship between Hubble rates can be obtained by making use of (53) and (54). One finds

$$\mathcal{H}^2 = \mathcal{H}_{\mathrm{h}}^2\left[1 - \frac{\mathcal{H}_{\mathrm{h}}^2 z^2}{2} + \frac{1}{16}\left(\mathcal{H}_{\mathrm{h}}^4 + \frac{4\mu}{a_{\mathrm{h}}^4}\right)z^4\right]^{-1}. \tag{64}$$

The map is schematically illustrated as

$$\begin{array}{ccc} d\tau^2 - a_{\rm h}^2 d\Omega_\kappa^2 & \xrightarrow{\tau\to\tilde{\tau}} & (1/n^2)d\tilde{\tau}^2 - a_{\rm h}^2 d\Omega_\kappa^2 \\ \downarrow z & & \downarrow z \\ n^2 d\tau^2 - a^2 d\Omega_\kappa^2 & \xrightarrow[\tau\to\tilde{\tau}]{} & d\tilde{\tau}^2 - a^2 d\Omega_\kappa^2 \end{array}$$

where τ and $\tilde{\tau}$ are the holographic and RSII synchronous times, respectively.

As an example, consider the RSII braneworld at $z_{\rm br} = \sqrt{2}\ell$ and the holographic braneworld at $z = 0$ with the corresponding Hubble rates $\mathcal{H}_{\rm br}^2$ and $\mathcal{H}_{\rm h}^2$. In Fig. 2 we plot $\mathcal{H}_{\rm br}^2$ versus $\mathcal{H}_{\rm h}^2$ for two values of the black hole mass parameter $\mu = 0$ (left panel) and $\mu\ell^4/a_{\rm h}^4 = 1/2$ with $z_{\rm br}^2/\ell^4 = 2$ (right panel). In both panels the shaded area, corresponding to the physical region $\rho_{\rm h} > 0$, is determined by the condition

$$2 - 2\sqrt{1 - \mu\ell^4/a_{\rm h}^4} \leq \mathcal{H}_{\rm h}^2\ell^2 \leq 2 + 2\sqrt{1 - \mu\ell^4/a_{\rm h}^4}. \qquad (65)$$

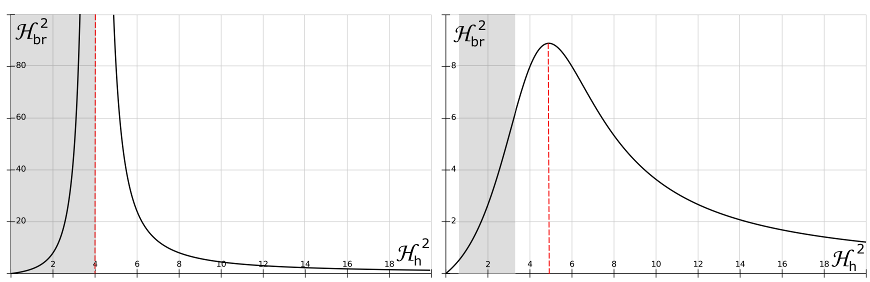

Figure 2: $\mathcal{H}_{\rm br}^2$ as a function of $\mathcal{H}_{\rm h}^2$ (both in units of $z_{\rm br}^{-2}$) defined by (64) for $\mu = 0$ (left panel) and $\mu\ell^4/a_{\rm h}^4 = 2$ with $z_{\rm br}^2/\ell^2 = 2$ (right panel). The region left from the vertical dashed red line is relevant for the one-sided version only. The shaded area corresponds to the physical region $\rho_{\rm h} > 0$.

6. Effective energy density

Next, we analyze a few special cases in two scenarios.

1. The RSII scenario with the primary braneworld at $z = z_{\rm br}$.
2. The holographic scenario with the primary cosmology on the AdS boundary at $z = 0$.

In each of the two scenarios we assume the presence of matter on the primary brane only and no matter in the bulk.

RSII scenario

In the RSII scenario the primary braneworld is the RSII brane at $z = z_{\rm br}$. The cosmology on the $z = 0$ brane emerges as a reflection of the RSII cosmology. For simplicity we take $z_{\rm br} = \ell$ and we fine tune the tension σ as in (14). Then, assuming the modified Friedmann equations (58) and (61) hold on the holographic brane, the effective energy density is given by

$$\frac{\rho_{\rm h}}{\sigma_0} = \frac{4\mathcal{E}(\rho/\sigma_0 + 1 - \mathcal{E})}{(\rho/\sigma_0 + 1 + \mathcal{E})^2 + \mu\ell^4/a^4}. \tag{66}$$

where $\mathcal{E}$ is defined by (63). Given the equation of state $p = p(\rho)$ on the RSII brane, the cosmological scale a is derived by integrating (22) and (24).

Thus, the two-sided model with positive energy density and positive μ maps into a holographic cosmology with negative effective energy density $\rho_{\rm h}$. For $\mu = 0$ the density$\rho_{\rm h}$ diverges with ρ as $1/\rho$. The one-sided model maps into two branches: $\mathcal{E} = -1$ branch identical with the two-sided map and the $\mathcal{E} = +1$ branch with a smooth positive function $\rho_{\rm h} = \rho_{\rm h}(\rho)$.

Holographic scenario

Suppose the cosmology on the $z = 0$ brane is known, i.e., the density $\rho_{\rm h}$, the pressure $p_{\rm h}$, and the cosmological scale $a_{\rm h}$ are known. If there is no matter in the bulk the induced cosmology on an arbitrary z-slice will be completely determined. Observers on the RSII brane on an arbitrary z-slice experience an emergent cosmology which is a reflection of the boundary cosmology. The general expression for the effective energy density ρ on the RSII brane is rather complicated but simplifies considerably for $z_{\rm br} = \ell$. In this case

$$\frac{\rho}{\sigma_0} = \left| \frac{1 + \rho_{\rm h}/\sigma_0 - \epsilon\sqrt{1 - 2\rho_{\rm h}/\sigma_0 - \mu\ell^4/a_{\rm h}^4}}{1 - \rho_{\rm h}/\sigma_0 - \epsilon\sqrt{1 - 2\rho_{\rm h}/\sigma_0 - \mu\ell^4/a_{\rm h}^4}} \right| - \frac{\sigma}{\sigma_0}, \tag{67}$$

where ϵ may take the values +1 or −1 . Hence, the function $\rho = \rho(\rho_{\rm h}, a_{\rm h})$ is not uniquely defined, although the mapping $a_{\rm h} \to a$ is unique (Fig. 3)

For an arbitrary $z_{\rm br} \neq \ell$ in the low density regime (relevant for the one sided version only), i.e., $\rho_{\rm h}^2 \ll \sigma_0^2$ and $\mu\ell^4 \ll a_{\rm h}^4$ we find:
a) For $\epsilon = -1$ at linear order in μ and quadratic order in $\rho_{\rm h}$ the effective energy density

$$\begin{aligned}\frac{\rho}{\sigma_0} &= 1 - \frac{\sigma}{\sigma_0} + \frac{z_{\rm br}^2}{\ell^2}\frac{\rho_{\rm h}}{\sigma_0} + \frac{1}{2}\frac{z_{\rm br}^2}{\ell^2}\left(\frac{z_{\rm br}^2}{\ell^2} + 1\right)\frac{\rho_{\rm h}^2}{\sigma_0^2} \\ &\quad - \frac{1}{2}\frac{z_{\rm br}^2}{\ell^2}\left(\frac{z_{\rm br}^2}{\ell^2} - 1\right)\frac{\mu\ell^4}{a_{\rm h}^4} + \dots .\end{aligned} \tag{68}$$

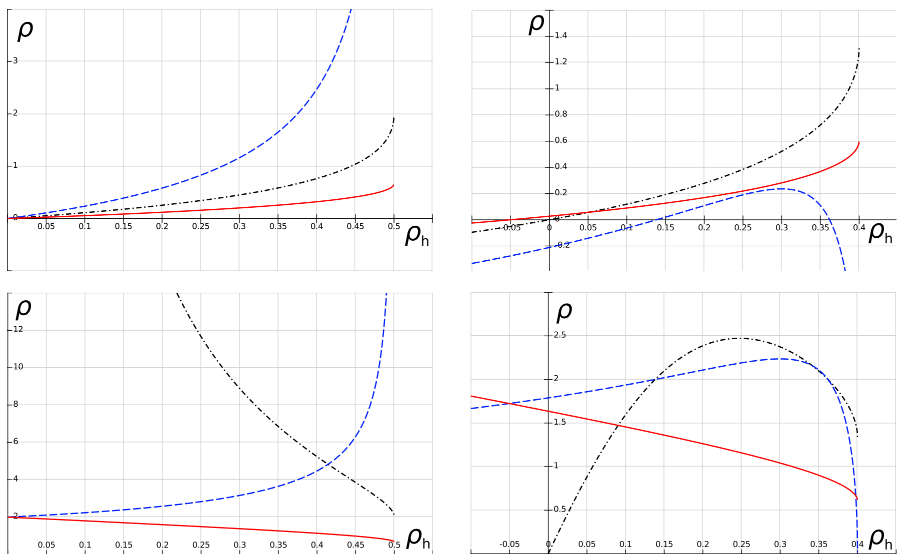

Figure 3: The effective density ρ on the RSII brane as a function of the density $\rho_{\rm h}$ on the holographic brane (both in units of σ_0) for $\sigma = \sigma_0$, $\epsilon = -1$ (top panels), $\epsilon = +1$ (bottom panels), and $\mu\ell^4/a_{\rm h}^4 = 0$ (left panels), 0.2 (right panels). The full red, dash-dotted black, and dashed blue lines represent $z_{\rm br}^2/\ell^2 = 0.5$, 1, and 2, respectively.

and pressure

$$p = -(\sigma_0 - \sigma) + \frac{z_{\rm br}^2}{\ell^2} p_{\rm h} + \dots . \tag{69}$$

Hence, at linear order the effective fluid on the RSII brane satisfies the same equation of state as the fluid on the holographic brane. The cosmological constant term will vanish on both branes if the RSII fine tuning condition is imposed.

b) For $\epsilon = +1$ at linear order

$$\frac{\rho}{\sigma_0} = \frac{z_{\rm br}^2/\ell^2 + 1}{z_{\rm br}^2/\ell^2 - 1} - \frac{\sigma}{\sigma_0} + \frac{z_{\rm br}^2/\ell^2}{(z_{\rm br}^2/\ell^2 - 1)^2}\frac{\rho_{\rm h}}{\sigma_0} - \frac{z_{\rm br}^2/\ell^2}{2(z_{\rm br}^2/\ell^2 - 1)^3}\frac{\mu\ell^4}{a_{\rm h}^4} + \dots . \tag{70}$$

Hence, in this case, the effective energy density ρ on the RSII brane differs from $\rho_{\rm h}$ on the holographic brane by a multiplicative constant and diverges in the limit $z_{\rm br} \to \ell$. The effective cosmological constant on the RSII brane does not vanish even if $\sigma = \sigma_0$ in which case

$$\Lambda_{\rm br} = \frac{6}{\ell^2}\frac{z_{\rm br}^2/\ell^2 + 1}{z_{\rm br}^2/\ell^2 - 1} - \frac{6}{\ell^2}. \tag{71}$$

7. Conclusions & Outlook

Our study can be summarized as follows:

- We have explicitly constructed the mapping between two cosmological braneworlds: holographic and RSII .
- The cosmologies are governed by the corresponding modified Friedmann equations.
- There is a clear distinction between 1-sided and 2-sided holographic map with respective 1-sided and 2-sided versions of RSII model.
- In the 2-sided map the low-density regime on the two-sided RSII brane corresponds to the high negative energy density on the holographic brane.
- The low density regime is maintained on both branes only in the one-sided RSII
- We have analyzed the effective energy density in two scenarios: the RSII scenario with the primary braneworld at $z = z_{\rm br}$ and the holographic scenario with the primary cosmology on the AdS boundary at $z = 0$. Then, in the holographic and RSII scenarios we will have emergent cosmologies on the RSII and holographic branes, respectively.

It is conceivable that we live in a braneworld with emergent cosmology. That is, dark energy and dark matter could be emergent phenomena induced by what happens on the primary braneworld. In this regards, the holographic scenario offers a few interesting possibilities. For example, suppose our universe is a one-sided RSII braneworld the cosmology of which is emergent in parallel with the primary holographic cosmology. If the energy density $\rho_{\rm h}$ on the holographic brane describes matter with the equation of state satisfying $3p_{\rm h} + \rho_{\rm h} > 0$, as for, e.g., cold dark matter, according to (70) and (71) we will have an asymptotically de Sitter universe on the RSII brane. If we choose the curvature radius ℓ so that the cosmological constant $\Lambda_{\rm br}$ fits the observed value, the quadratic term will be comparable with the linear term today but will strongly dominate in the past and hence will spoil the standard cosmology. However, the standard ΛCDM cosmology could be recovered by including a negative cosmological constant term in $\rho_{\rm h}$ and $p_{\rm h}$ and fine tune it to cancel $\Lambda_{\rm br}$ up to a small phenomenologically acceptable contribution.

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