Cycles cohomology by integral transforms in derived geometry to ramified field theory*

Prof. Dr. Francisco Bulnes†
Head of Research Department in Mathematics and Engineering,
TESCHA, IINAMEI Director

Abstract

Several geometrical Langlands correspondences are considered to determine equivalences necessary to the obtaining in the quantized context from differential operators algebra (actions of the algebra on modules) and the holomorphic bundles in the lines bundle stacks required to the modeling of the elements of the different physical stacks and the extension of their field ramifications to the meromorphic case. In this point, is obtained a result that establish a commutative diagram of rings and their spectrum functor involving the non-commutative Hodge theory, and using integral transforms to establish the decedent isomorphisms in the context of the geometrical stacks to a good Opers, level. The co-cycles obtained through integral transforms are elements of the corresponding deformed category to mentioned different physical stacks (where are had, even field singularities). In this point, a justification on the nature of our twisted derived categories and their elements as ramifications of a field (to the field equations) is the followed through the Yoneda algebra where is searched extends the action of certain endomorphisms of Verma modules of critical level through the Lie algebra of ramifications, whose cohomological space has zero dimension. This establishes solution classes to the QFT-equations in field theory through the spectrum of their corresponding differential operators.

Key words: Deformed Derived Categories, Field Ramifications, Geometrical Langlands Correspondences, Generalized Verma Modules, Grothendieck Schemes, Moduli Stacks, Integral Transforms, Spec Functor, Twisted Derived Categories, Yoneda Algebra, QFT-equations, “Quantized” Differential Operators Algebra.

AMS 2010 Classification. 53D37; 11R39; 14D24; 83C60; 11S15

---

* Post-Doctoral Research supported by the State of Mexico Government. Presented talk in the 9th MATHEMATICAL PHYSICS MEETING: School and Conference on Modern Mathematical Physics, in the Physics Institute of Belgrade City, Serbia, 2017.

† e-mail address: francisco.bulnes@tesch.edu.mx

1. Introduction and General Discussion.

Equivalence classes on field solutions to establish an Universe theory involving the QFT, TFT, SUSY and the inclusion of the strings theory in all their modalities (for examples the heterotic strings) are obtained. Then through corresponding sheaves of differential operators of the field equations; sheaves of coherent D-modules are identified [1].

This identification will serve to build, on the equivalences basis of the corresponding Zuckerman functor, the universal functors on derived categories of Harish-Chandra modules to the geometrical Langlands programme in mirror theory [2], [3]. Likewise, the incorporation of the geometrical Langlands ramifications will establish extensions of a connection beyond the holomorphicity to much of the vector complex bundles that can be constructed to a large stack of physical phenomena in the search to obtain solutions of field through the curvature and torsion tensors of the space-time.

The obtained development includes D-modules complex of infinite dimension, generalizing for this via, to the BRST-cohomology. Then the integrable systems class can be extended in mathematical-physics and with it the possibility of obtaining of a general theory of integral transforms to the study of the space-time (cohomology of integral operators [4]) considering the kernels of the germs of the corresponding sheaves to the vector complex bundles, and then the measure of much of their field observables [3].

Having these Langlands correspondences we can to tend a bridge to complete a classification of the different operators to the field equations using on the base the Verma modules that are classifying spaces, the differential operators of $SO(1, n+1)$, where elements of the Lie algebra $\mathfrak{sl}(1, n+1)$, are differential operators of the modern mathematical physics [1]. The cosmological problem that exist is to reduce the number of the field equations that are resoluble under the same gauge field (Verma modules) and extend the gauge solutions to other fields using the symmetries of topological groups that define their interactions.

This extension can be given for a global Langlands correspondence between categories of Hecke sheaves on an adequate moduli stack and the category of holomorphic ${}^{L}G$- bundles with a special connection (Deligne connection). The corresponding D-modules can be viewed as sheaves of conformal blocks (or co-invariants) being images under a version of the generalized Penrose transform [1], [6], naturally arising in the frame of the conformal field theory.

Finally we can to determine "quantized versions" of the derived categories and their cohomology.

2. Background Results.

We need a scheme theory where we can exhibit the image in the context of commutative rings of the corresponding spectrum functor of the derived category of the sheaves of the field equations germs to Lie algebra $\hat{\mathfrak{g}}$, of the connection of the corresponding holomorphic bundle.

Theorem 2. 1 (F. Bulnes) [7]. If we consider the category $M_{\mathcal{K}_{\mathcal{F}}}(\hat{\mathfrak{g}}, Y)$, then a scheme of their spectrum $\mathrm{V}_{critical}^{Def}$, where Y, is a Calabi-Yau manifold comes given as:

$$Hom_{\hat{\mathfrak{g}}}(X, V_{critical}^{Def}) \cong Hom_{Loc_{L_G}}(V_{critical}, M_{\mathcal{K}_F}(\hat{\mathfrak{g}}, Y)), \tag{1}$$

Proof. [7].▪

Then we can to establish the following results considering the moduli problems between objects of an algebra A_∞, and objects of a vector bundle.

Studies realized using commutative rings extended by the Yoneda algebra establish the existence inside the moduli problems of an isomorphism between functors Ext with deformations in the moduli problem defined for Loc_{L_G}, and spectrum, the corresponding deformed derived category $\mathcal{D}^s(Bun_G)$. Likewise we have the following result.

Theorem. 2.2. The Yoneda algebra $\mathrm{Ext}_{\mathcal{D}^s(Bun_G)}(\mathcal{D}^s, \mathcal{D}^s)$, is abstractly A_∞, -isomorphic to $\mathrm{Ext}^\bullet_{Loc_{L_G}}(\mathcal{O}_{Op_{L_G}}, \mathcal{O}_{Op_{L_G}})$.

Proof. [8],[9].▪

This result bring in particular that formal deformations of the sheaf $\mathcal{D}^s$, can be consigned in $\mathcal{D}^s_{Bun_G}$ -mod, which in the moduli stack language can be re-written using the theorem 2.1, as

$$Spec_{Bun_C} TBun_G = T^\vee Bun_G, \tag{2}$$

We can establish a long sequence where the correspondence between moduli stacks and cohomological classes as products of the generalized Verma modules (see table [10]) can be given until critical level of the vector holomorphic bundle. But is here where precisely the cohomological space $H^\bullet(T^\vee Bun_G, \mathcal{O}))$, has their corresponding version with coefficients in the Verma module at critical level [1] as $H^\bullet(\mathfrak{g}[[t]], \mathfrak{g}; \mathbb{V}_{crit}))$. Of fact, this appears inside moduli identity given in [11], using the critical level bundle

[1] $\mathbb{V}_{crit} = U_{crit}\hat{\mathfrak{g}} \otimes_{\mathfrak{g}[[z]]} \mathbb{C}$.

$K^{1/2}$.

The integration process is given or described for the main theorem in [12], which was demonstrated with detail.

3. Main Result.

We establish the following hypothesis using the before results as platform and as part of this hypothesis. Likewise, to define an adequate moduli problem we use the scheme of the Grothendieck type given by (1), and to extension of commutative rings, we use the theorem 2. 2.

The important prospective of the theorem 2. 2., that we need, is the fact that formal deformations of sheaves can be extended to deformation of categories in QFT.

But also, as was mentioned in the before section, the twisted nature of the derived categories $\mathcal{D}^{\times}$, must be done on an appropriated stack (which as we know must be on an appropriated vector bundle).

Theorem 3. 1. One meromorphic extension of one flat connection given through a Hitchin construction we can give the following commutative cocycles diagram to the category $M_{\mathcal{K}_{\mathcal{F}}}(\hat{\mathfrak{g}}, Y)$,

$$\begin{array}{ccccc} \mathbf{h}\epsilon H^0(T^\vee Bun_G, \mathcal{D}^s) & \xrightarrow{d} & H^1(T^\vee Bun_G, \mathcal{O}) & \xrightarrow{\cong} & \Omega^1[\mathbf{H}] \\ \cong\downarrow & & \cong_{\Phi\mu}\downarrow & & \downarrow\pi \\ a\epsilon\mathbb{C}[Op_{L_G}] & \xrightarrow{d} & \Omega^1[\mathcal{O}_{Op_{L_G}}] & \xrightarrow{d} & C\times B \end{array} \qquad (3)$$

Their demonstration was given with detail in [13]. However, we will give some sketch and we discuss some key steps and conclusions from their demonstration.

From the lemma 3. 1 of [11], and using some arguments inside the demonstration of the main theorem given in [12], the cohomology space $H^\bullet(?, \Omega^\bullet)$, is exhibed as the space $H^\bullet(H^\vee, \Omega^\bullet)$, inside the quasi-coherent category given by $M_{\mathcal{K}_{\mathcal{F}}}(\hat{\mathfrak{g}}, Y)$, which carry us to the ramification problem. But we consider QFT and TFT, in the derived categories framework to define cocycles of $M_{\mathcal{K}_{\mathcal{F}}}(\hat{\mathfrak{g}}, Y)$

To the demonstration of the theorem 3. 1, firstly was demonstrated the equality between $R^1\chi_*(\mathcal{D}^s)$ and $R^1\chi_*(\mathbf{h}) \in \Omega^1[\mathbf{H}]$ [2] in the derived class $\mathcal{D}^\times$. Then we consider a Langlands correspondence such that:

$$\Phi^i(\mathfrak{c}(\mathcal{O}_{\mathbb{V}})) = \mathfrak{c}(\mathcal{O}_{\mathbb{V}}) \boxtimes \wedge^i \mathbb{V}, \tag{4}$$

arriving to $\Omega^i[Op_{^LG}]$, which gives the equivalence of complexes

$$\{d\mathbf{h} = 0\} \cong {}^L\Phi^\mu \{da = 0\}, \tag{5}$$

which requires the correspondence

$$\tilde{\mathfrak{c}} : D_{Coh}(T^\vee Bun_G, \mathcal{O}) \cong D_{Coh}(^L T^\vee Bun_G, \mathcal{O}), \tag{6}$$

Then is proved the first descendent isomorphism of (3).

By the Frenkel equivalence $D^b(\mathfrak{g}_{\mathcal{KC}} - mod_{nilp})^{I_0} \cong D^b(\mathcal{Q}Coh(MOp_{^LG}^{nilp}))$, each quasi-coherent sheaf on the kernel of the right side of this equivalence corresponds an object of $D^b(M_{\mathcal{K}_\mathcal{F}}(\hat{\mathfrak{g}}, Y))^{I_0}$. Then the functor $\Phi_{\mathcal{O}_{Spu}^{DG}}$, is a Hecke functor. And also, their integral transform is such that h : $Bun_{Higgs} \to B$ (to a quantized) and their image is equivalent to $D_{coh}(^L Bun, \mathcal{D})$. Then the geometrical Langlands conjecture in terms of Higgs bundles consider a functor between the categories $D_{Coh}(^L Loc, \mathcal{O})$, with the action of the Hecke functors on $D_{Coh}(^L Bun, \mathcal{D})$. But $MOp_{^LG}^{nilp} \cong Op_{^LG}^{nilp} \times \tilde{\tilde{\mathcal{N}}} \;/^L\; G$, by the Steinberg manifold structure to the Langlands correspondence given by $\mathfrak{c}$, such that $\tilde{\mathfrak{c}} = \mathfrak{c}(\mathcal{O}_{Sp_uC}(\mathbb{V}) \times \mathbb{C}^\times)$, where $\mathcal{O}_{Sp_uC} = \mathcal{O}_{\tilde{\mathcal{N}}} \otimes_{\mathcal{O}\mathcal{N}} \mathbb{C}$, and $\mathcal{N} \subset^{\mathcal{L}} \mathfrak{g}$. By K-theory, the Steinberg manifold have elements $C \times B$, that satisfy $Isom(d\mathbf{h}) = d(da), \forall a \in \mathbb{C}[Op_{L_G}(D)]$. Then is had that

$$d(da) \overset{^L\Phi^\mu}{\longleftrightarrow} Isom d\mathbf{h}, \tag{7}$$

That is to say, the kernels of $\Omega^i, i = 1, 2, \ldots,$ are the that are in the sheaf

[2] The first differential in a spectral sequence $H^*(\mathcal{D})[[s]]$ implies $H^*(\mathcal{D}^s)$, for the deformation $\mathcal{D} \to \mathcal{D}^s$.

$\mathcal{O}_{Op_{L_G}}$, [3] that is to say, there is an extended Penrose transform such that their kernel set has as elements the fields $\mathbf{h}$, with $\mathrm{Isom}(d\mathbf{h}) = 0$, in the hyper-cohomology. Then in hyper-cohomology, in the down line and using the Hitchin mapping the fields a, are those that satisfy $d(da) = 0$, in the hyper-cohomology $\mathbb{H}(\Omega^1 \xrightarrow{d} \Omega^2 \xrightarrow{d} \dots)$.

In the context of the differential operators algebra we can give the commutative rings diagram:

$$\begin{array}{ccccc} H^\bullet(\mathfrak{g}[[z]], \mathbb{V}_{critical}) & \rightarrow & H^\bullet(\mathfrak{g}[[z]], \mathfrak{g}, \mathbb{V}_{critical}) & \xrightarrow{\cong} & \Omega^\bullet[\mathbf{H}] \\ \cong\downarrow & & \cong_\Phi\downarrow & & \downarrow \pi \\ \mathbb{C}[Op_{L_G}] \xrightarrow{d} & & \Omega^\bullet[Op_{L_G}] \xrightarrow{d} & & \mathbf{H}^\vee \end{array} \tag{8}$$

where $\mathbf{H}^\vee = Spec_H SymT[Op_{L_G}(D)]$. Then we have the Penrose transform in the descendent isomorphism whose field solutions are to the equations $d(da) = 0$. An extended Penrose transform to the deformed modules version $H^\bullet(\mathfrak{g}[[z]], \mathfrak{g}, \mathbb{V}_{critical})$, consider the Fourier-Mukai transform. Then the Yoneda algebra given by $\mathrm{Ext}_{\mathcal{D}^s(Bun_G)}(\mathcal{D}^s, \mathcal{D}^s)$, can to establish the endomorphism of critical level modules. Then is completed the sequence of critical Verma modules (projective Harish Chandra module to whole the sequence). Likewise the global functor to the diagram in question until $\Omega^\bullet[Op_{L_G}]$, is ${}^L\Phi^\mu(\mathcal{M}) = \mathcal{M} \boxtimes \rho^\mu(\mathbb{V})$ with ${}^L\Phi^\mu$ is the Hecke functor. Finally to obtain the spectrum (in a Hamiltonian variety) $\mathbf{H}$, we use the quantized version of the corresponding cohomology space $H^q(Bun_G, \mathcal{D}^s) = \mathbb{H}^q_{G[[z]]}(\mathbf{G}, (\wedge^\bullet[\Sigma^0] \otimes \mathbb{V}_{critical}; \partial))$. Finally and after of a good work in the graded vector spaces in the deformed derived category $\mathrm{D}_{coh}({}^LBun, \mathcal{D}^\times)$, which are included in the quasi-coherent category $M_{\mathcal{K}_F}(\hat{\mathfrak{g}}, Y)$, we have $Spec_G^{\mathfrak{g}[[z]]/G}(\Omega^1(\mathbf{H})) = Y$.

4. Application (Gravitational waves as oscillations in the space-time curvature/spin).

We consider now $G = GL(1, \mathbb{C}) \cong^L G$. We consider the bundle stack given for [4]

[3] $H^\bullet Bun_G, \mathcal{O}) \cong \mathcal{O}_{Op_{L_G}} = Ker(U, \nabla)$, where ∇, induces a holomorphic connection on lines bundles.

[4] Here $Pic(C)$, is the moduli stack Bun.

$$\mathbb{M} = Pic(C), \tag{9}$$

which is known as Picard variety of C. Then the Hecke functor is the mapping

$$\Phi^1 : D_{coh}(Pic(C), \mathcal{D}) \to D_{coh}(C \times Pic(C), \mathcal{D}), \tag{10}$$

which is pull-back of the Abel-Jacobi mapping:

$$aj : C \times Pic^d(C) \to Pic^{d+1}(C), \tag{11}$$

with correspondence rule

$$(L, \chi) \mapsto L(\chi), \tag{12}$$

In this case, the geometrical Langlands correspondence comes give as:

$$\mathfrak{c}(\mathbb{L}) = \left\{ \begin{array}{c} \textit{The unique translation invariant rank on local system} \\ \textit{on variety } Pic(C), \textit{ whose restriction on each component} \\ Pic^d(C), \textit{ has the same monodromy as } \mathbb{L}. \end{array} \right\} \tag{13}$$

where $\mathbb{L}$, is the space of the Langlands data (bundle and connection) (L, ∇), that is a rank one local system on C. Due to that $\pi_1(Pic^d(C))$, is the abelianization of $\pi_1(C)$, and the monodromy of the space $\mathbb{L}$, is Abelian, we can view this space $\mathbb{L}$, as a local system on each component $Pic^d(C)$, of $Pic(C)$.

Them likewise, considering the pull-back of the local system $\mathbb{L}$, to the various factors of the $d-th$ Cartesian power $C^{\times d}$, of C, and tensor of these pull-backs to get rank one local system $\mathbb{L}^{\boxtimes d}$, where $\boxtimes$, is a micro-local tensor product.From a point of view of the field equations, each component of the correspondences space $\mathfrak{c}(\mathbb{L})$, on $Pic^d(C)$, a trace of particles in the symplectic geometry that can be characterized in a Hamiltonian manifold, with the due quantization of the coherent sheaves of the differential operators of the field equations.

Likewise, using a Hitchin′s abelianization we can induce the geometrical Langlands correspondence $\mathfrak{c}$, as was planted to the case of the group $G = GL(n, \mathbb{C})$, considering the correspondence $\mathfrak{c}$, as:

$$\mathfrak{c} = quant_{Bun} \circ \Phi \circ quant_C^{-1}, \tag{14}$$

where Φ, is the Fourier-Mukai transform defined to this case as:

$$\Phi : D_{coh}(quant_{Bun}, \mathcal{D}) \to D_{coh}(C \times quant_{Bun}, \mathcal{D}), \tag{15}$$

Here, the quantization procedures $quant_{Bun}$, and $quant_C$, are appropriately understood non-Abelian Hodge correspondences. An adequate Hitchin mapping can give solution to the equations through Hamiltonian states, in the non-Abelian context of the Hodge theory [13] in hypercohomology:

$$d(da) = 0, \tag{16}$$

$\forall a \in \mathbb{C}[Op_{L_G}(D)]$, having as integral the integral transforms composition:

$$\mathfrak{c} \circ \Phi^{\mu} =^{L} \Phi^{\mu} \tag{17}$$

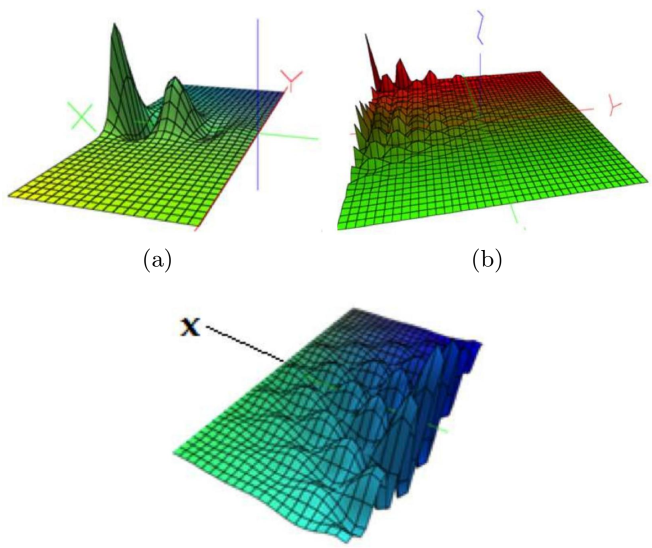
where the states of the quantum field are the cotangent vector (Higgs fields) $\mathbf{h}$, such that

$$Isomd\mathbf{h} = 0, \tag{18}$$

Then by superposing of these states considering the field corresponding ramifications, we have:

$$\mathcal{H} = H^0(\omega_C) \oplus H^0(\omega_C^{\otimes 2}) \oplus \cdots \oplus H^0(\omega_C^{\otimes n}), \tag{19}$$

which has their re-interpretation as the curvature energy expressed through the H-states which can be written using the superposing principle to each connection $\omega_C^{\otimes j}$, (with C, a curve) that describes the corresponding dilaton (photon). Likewise, in a Hamiltonian densities space [14] we have the figure 1A, considering a Hitchin base.



(a) (b)

(c)

Figure 1: a). Direct sum of H-states to establish the curvature measure by field ramification. b). The waves that are spinor waves which can be consigned in oscillations in the space-time in the presence of curvature to the change of particles spin. This also corresponds to quantum gravity model. c). Gravitational waves produced by quantum gravity due the H-states on a cylindrical surface. Their propagation is realized on axis X. These gravitational waves are originated for the oscillations in the space-time-curvature/spin (that is to say using causal fermions systems).

In the case of spinor representation the corresponding H-states can be given as spinor waves (see figure B) which can be consigned in oscillations in the space-time-curvature/spin, to a microscopic deformation measured in $\mathcal{H}$.

Specialized Notation

$\mathcal{D}^{\times}$ -Twisted sheaf of differential operators to our Oper, given by $\mathrm{Loc}_{^LG}(\mathcal{D}^{\times})$.

$\mathrm{K}^{1/2}$ - Root square of the canonical line bundle on Bun_G, corresponding to the critical level. This is a divisor vector bundle.

$\mathrm{Bun}_G(X)$ -Category of principal G - bundles over $C \times X$. Also is the moduli stack of principal G -bundles over C.

$\mathrm{Loc}_{^LG}(\mathcal{D}^{\times})$ -Set of equivalence classes of ^LG -bundles with a connection on $\mathcal{D}^{\times}$. This space shape a bijection with the set of gauge equivalence classes of the ramified operators, as defined in [14],[15].

$Pic(C)$- Picard variety of C.

H− Hamiltonian variety. Space of the function of H-states. Their manifestation are energy waves corresponding to the connection of the level field equation.

References

[1] F. Bulnes, Cohomology of Moduli Spaces in Differential Operators Classification to the Field Theory (II), in: Proceedings of Function Spaces, Differential Operators and Non-linear Analysis., 2011, Tabarz Thur, Germany, Vol. 1 (12) pp001-022.

[2] F. Bulnes, Penrose Transform on Induced DG/H-Modules and Their Moduli Stacks in the Field Theory, Advances in Pure Mathematics 3 (2) (2013) 246-253. doi: 10.4236/apm.2013.32035.

[3] A. Kapustin, M. Kreuser and K. G. Schlesinger, Homological mirror symmetry: New Developments and Perspectives, Springer. Berlin, Heidelberg, 2009.

[4] F. Bulnes, Integral geometry and complex integral operators cohomology in field theory on space-time, in: Proceedings of 1st International Congress of Applied Mathematics-UPVT (Mexico)., 2009, vol. 1, Government of State of Mexico, pp. 42-51.

[5] F. Bulnes, M. Shapiro, On general theory of integral operators to analysis and geometry, IM-UNAM, SEPI-IPN, Monograph in Mathematics, 1st ed., J. P. Cladwell, Ed. Mexico: 2007.

[6] F. Bulnes, R. Goborov, Integral Geometry and Complex Space-Time Cohomology in Field Theory, Pure and Applied Mathematics Journal. Special Issue:Integral Geometry Methods on Derived Categories in the Geometrical Langlands Program. Vol. 3, No. 6-2, 2014, pp. 30-37. doi: 10.11648/j.pamj.s.2014030602.16.

[7] F. Bulnes, Integral Geometry Methods on Deformed Categories in Field Theory II, Pure and Applied Mathematics Journal. Special Issue: Integral Geometry Methods on Derived Categories in the Geometrical Langlands Program, 3(6-12), (2014), 1-5, doi: 10.11648/j.pamj.s.2014030602.11.

[8] N. Yoneda, On the homology theory of modules, J. Fac. Sci. Univ. Tokyo. Sect. I, 7, (1954), 193227.

[9] E. Frenkel, C. Teleman, Self extensions of Verma modules and differential forms on opers, Comps. Math., 142, (2006), 477-500.

[10] F. Bulnes, Geometrical Langlands Ramifications and Differential Operators Classification by Coherent D-Modules in Field Theory, Journal of Mathematics and System Sciences, 3(10), (2013), 491-507.

[11] F. Bulnes, Integral geometry methods on deformed categories to geometrical Langlands ramifications in field theory, Ilirias Journal of Mathematics, 3(1), 1-13.

[12] F. Bulnes, Moduli Identities and Cycles Cohomologies by Integral Transforms in Derived Geometry, Theoretical Mathematics and Applications, 6(4), (2016), 1-12.

[13] F. Bulnes, Extended d Cohomology and Integral Transforms in Derived Geometry to QFT-equations Solutions using Langlands Correspondences, Theoretical Mathematics and Applications, Vol. 7 (2), pp51-62.

[14] F. Bulnes, Integral Geometry Methods in the Geometrical Langlands Program, SCIRP, USA, 2016.

[15] A. Abbondandolo, M. Schwarz, Floer homology of cotangent bundle and the loop product, Geom. Top., 14(3), (2010), 1569-1722.