Supersymmetric path to unification of gravity with particle physics*

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Abstract

It has been showed recently that spin curves space along with mass and ultra-high spin/mass ratio of the elementary particles makes Einstein's gravity strong, shifting gravitational influence from Planck to the Compton scale. Although this increases conflict between gravity and quantum theory, the compatibility of the spinning Kerr-Newman (KN) gravity with electroweak quantum particles can be achieved by using the supersymmetric Higgs model, which creates a free from gravity supersymmetric and superconducting core of the particle. The corresponding BPS-saturated solution to generalized LG model takes the form of a supersymmetric bag model, which provides a flat supersymmetric vacuum state inside the bag. The bag is deformable, and its shape is controlled by supersymmetry providing compatibility of the core with external gravitational and electromagnetic (EM) field. In particular, for the spinning KN gravity bag takes the form of oblate disk with a circular string placed on the disk border. Excitations of the KN EM field create circular traveling waves. The super-bag solution is upgraded to the Wess-Zumino supersymmetric QED model, indicating a bridge from the super-bag model to perturbative formalism of the conventional QED.

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1. Introduction

Modern physics is based on Quantum theory and Gravity. The both theories are confirmed experimentally with great precision. Nevertheless, they are contradicting and cannot be combined in a unified theory. This contradiction is mutual. On the one hand, gravitational field cannot be quantized, on the other hand, the stochastic representation of the quantum particles as point-like objects controlled by the wave function is not suitable for gravity, which requires representation in terms of the real physical fields generating the right hand side of the Einstein equations, $G_{\mu\nu} = 8\pi T_{\mu\nu}$.

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Treatment of the quantum particles as the extended semiclassical objects was started in 1970's in the models of solitons, strings and bag models, and superstring theory was considered to be the most promising among them. However, as mentioned John Schwarz, *"...Since 1974 superstring theory stopped to be considered as particle physics... "* and *"... a realistic model of elementary particles still appears to be a distant dream ... "* [1]). One of the reasons of this is that extra dimensions are compactified with extra tiny radii of order the Planck length 10^{-33} cm, which does not correlate with characteristic lengths of quantum physics and makes impossible to test extra dimensions with currently available energies. The idea to bring fundamental gravitational scale close to the weak scale was considered in different approaches, and in particular, in the brane world scenario, where the weakness of the localized 4d gravity is explained by its "leaks" into the higher-dimensional bulk, and the brane world mechanism allowed to realize ideas of the superstring theory for any numbers of the extra dimensions [2].

Alternative ideas were related with nonperturbative 4D solutions of the non-linear field models – solitons, in particular, solitonic solutions to low energy string theory [3, 4, 5, 6]. This approach, being nonperturbative, is akin to the Higgs mechanism of symmetry breaking and linked with nonperturbative approach to electroweak sector of the Standard Model. The most known is the Nielsen-Olesen model of dual string based on the Landau-Ginzburg field model for a phase transition in superconducting media, and also the famous MIT and SLAC bag models [7, 8, 9] which are similar to solitons, but being soft, deformable and oscillating, acquire many properties of the dual string models. Besides, being suggested for confinement of quarks, the bag models assume consistent implementation of the Dirac equation. The question on consistency with gravity is not discussed usually for the solitonic models, as it is conventionally assumed that gravity is very weak and is not essential on the scale of electroweak interactions. For example, in [10] we read *"... quantum gravity effects are usually very small, due to the weakness of gravity relative to other forces. Because the effects of gravity are proportional to the mass, or the energy of the particle, they grow at high energies. At energies of the order of E* 10^{19} *GeV, gravity would have a strength comparable with that of the other Standard Model interactions."*

Meanwhile, in a relativistic theory, spin is not separable from rotation, and its influence on metric should also be taken into account. Analyzing metric of the Kerr-Newman (KN) rotating black hole solution with parameters of an electron, we obtain strong influence of spin and recognize that the usual assumption on the weakness of gravity is not correct. Indeed, nobody says that gravity is weak in Cosmology where physics is determined by giant masses. Similarly, the giant spin/mass ratio of the elementary particles, $J/m = 10^{20} - 10^{22}$ (in the dimensionless Planck's units $G = c = \hbar = 1$) shows that the energy equivalent to unit of quantum spin on the Planck scale must be the unit of Planck mass $E_P = M_P = 1$, leading to the ratio $J/m \approx M_P/m \sim 10^{20}$. It shows that giant spin of particles must have a giant impact on the metric. The commonly accepted view that gravity is weak and not essential in particle physics becomes not compatible with Planck scale of gravitational interaction, and should be replaced by principally new point of view that scale of gravitational interaction is to be shifted about 20 orders of magnitude to the Compton lengths [11, 12, 13]. Influence of spin on metric becomes extreme strong, and even crucial for the electroweak interactions.

Analysis of the Kerr-Newman (KN) solution with parameters of an electron confirms this statement, showing that space-time is strongly deformed in the Compton zone, contrary to the usually accepted Planck length, and reason of this deformation is that spin deforms space along with mass. In other words, *spin is gravitating*, and ultra-high spin/mass ration of the electron, $a = J/m \sim 10^{22}$, breaks space topologically, creating the naked singular ring and two-sheeted space, which differs from Minkowski space so strongly that neither Dirac theory nor perturbative QED can be applied.

This great influence of spin, leading to drastic shifts of the scale of gravitational interaction was not taken into account before, and apparently, it was the main reason of the failure previous attempts of unification of gravity with particle physics. So large shifts of the scale of interaction creates a new paradigm and requires a principally new approach, in which gravity is not subordinated to quantum theory, and both these theories impact on process of unification on an equal footing.

In this article we suggest a solution of the problem of unification with gravity based on this new paradigm. We show that conflict between gravity and quantum theory can be resolved *without modification of the Einstein-Maxwell gravity – the spoiled by gravity space can be cured by a supersymmetric bag model*, in which the singular region of KN solution is replaced by the flat internal space of the Compton size. We find the corresponding non-perturbative BPS-saturated solution in frame of the supersymmetric generalized Landau-Ginzburg model, in which boundary of the bag is formed by a domain wall interpolating between the external KN gravity and the supersymmetric vacuum state inside the bag. Similar to the usual bag models, the super-bag model is deformable and displays a super-consistency with the external gravitational and electromagnetic KN field, in the sense that its shape and dynamics are fully defined by matching its boundary with a special surface (which can be called as "zero gravity surface"), where the external gravitational field is compensated by electromagnetic field. This surface determines position of the domain wall, and therefore, it determines shape of the bag, which gets a disk-like configuration with a closed string lying along sharp border of the disk [14, 15, 16, 17]. Therefore, this new concept leads us again to a string theory, which is however, four-dimensional, and differs essentially from the famous superstring theory based on compactification of higher dimensions at Planck scale.

One of the main and the most studied quantum particles is an electron, and assuming that dressed electron has such a SuperBag structure we have to explain first of all, while this structure was not observed experimentally, and second, to put out the possible relationship with the Dirac theory of electron and with perturbative QED. The first signal on the relation of the KN solution with Dirac electron was obtained by Carter [18, 19], who obtained that KN solution has gyromagnetic ratio $g = 2$ as that of the Dirac electron. Besides, one of the basic features of the Bag models is compatibility with the Dirac equation [7, 8], which in the Kerr geometry was supported by two aligned spinor solutions given by the Kerr theorem, [20, 14]. The link with QED is much more problematic, and we show here that the supersymmetric Landau-Ginzburg model can naturally be upgraded to the Wess-Zumino SuperQED model [21], revealing connections of the non-perturbative SuperBag solution with the conventional perturbative technics used in QED.

2. Super-bag model as BPS solution to generalized LG model

2.1. Basic features of the ultra-rotating Kerr-Newman solution

It has been recently obtained [22, 23], that the source of ultra-spinning Kerr-Newman (KN) solution can be considered as a superconducting soliton having many features of the bag model [14, 15, 24], but with the essential advantage of compatibility with Einstein-Maxwell gravity in four dimensions. As is known, the bag models take intermediate position between strings and solitons [25, 26, 27]. Although, the bags were initially offered as the extended models of hadrons, [7, 8, 9], being based on the Abelian Higgs model of symmetry breaking their indicated rather applicability to the Salam-Weinberg model of leptons, which was one of the reasons to consider the gravitating KN bag as the model for consistent with gravity leptons.

The spinning KN solution is of particular interest in this regard, since, as it was obtained by Carter [19, 18], that gyromagnetic ratio of the KN solution is $g = 2$, and therefore corresponds to the external field of the electron. The spin/mass ratio of the electron is about 10^{22}, and structure of source of the KN solution for such a huge spin should shed the light on origin of the conflict between gravity and quantum theory. One can see that the KN field with parameters of electron becomes extremely strong on the Compton distances, so that the BH horizons disappear and the Kerr singular ring of the Compton radius $a = \hbar/m$ becomes open, which breaks topology of space-time and creates two-sheeted metric.
In the Kerr-Schild approach, metric of the KN solutions is [19]

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \tag{1}$$

where $\eta_{\mu\nu}$ is metric of an auxiliary Minkowski space M^4, (signature $(-+++)$), and H is the scalar function which for the KN solution takes the form

$$H_{KN} = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}, \tag{2}$$

where r and θ are oblate spheroidal coordinates, and k_μ is a null vector field $k_\mu k^\mu = 0$, forming a Kerr congruence – the vortex of polarization of gravitational and electromagnetic field in the Kerr space-time, see Fig.1.

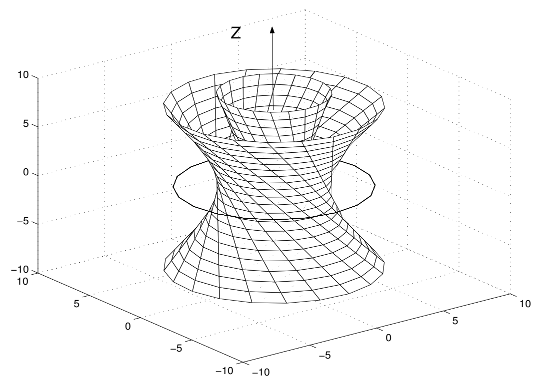

Figure 1: Vortex of the Kerr light-like (null) congruence k^μ propagates analytically from negative sheet of Kerr metric, $r < 0$, to positive one, $r > 0$. In the equatorial plane, $\cos\theta = 0$, the Kerr congruence is focused on the Kerr singular ring, $r = \cos\theta = 0$.

The Kerr singular ring corresponds to border of the disk $r = 0$, in the equatorial plane $\cos\theta = 0$.

Similarly, vector potential of KN solution is also collinear with the null direction k_μ,

$$A_\mu = -\frac{er}{(r^2 + a^2\cos^2\theta)} k_\mu \tag{3}$$

The KN metric becomes two-sheeted, since the Kerr congruence

$$k_\mu dx^\mu = dr - dt - a\sin^2\theta d\phi, \tag{4}$$

is out-going at the 'positive' sheet of the metric, $r > 0$, and passes analytically to 'negative' sheet, $r < 0$, being extended via ring $r = 0$, where it becomes in-going. The two null vector fields $k_\mu(x)^\pm$ become different at $r > 0$ and $r < 0$, leading to two different metrics $g^\pm_{\mu\nu} = \eta_{\mu\nu} + 2Hk^\pm_\mu k^\pm_\nu$ on the positive and negative sheet of the same Minkowski background. Similarly, it leads also to two-sheeted vector-potential $A^\pm_\mu$, that makes space inappropriate for quantum theory, and therefore, conflict between quantum theory and gravity is shifted by 22 orders earlier then it is usually expected, from the Planck to the Compton scale. As usually, singularity is signal to new physics – theory of more high level. The KN gravitational field is strong near the Kerr singular ring and creates vortex of the space-time polarization in the Compton zone of the dressed electron, which should be flat for work of quantum theory. It is usually assumed that in vicinity of strong field, gravity should be modified to a new Quantum Gravity. Taking into account sharp incompatibility of Quantum and Gravity, natural requirement for such new theory would be separation of their zones of influence: formation of the internal zone

(I) – flat core for quantum theory, and external zone

(E) – for undisturbed gravitational and electromagnetic fields.

There should also be selected intermediate zone

(R) – interpolating between **(I)** and **(E)**.

In the case of strong KN field, these demands become so restrictive that determine structure of the new theory almost uniquely. It turns out that the flat Compton zone free from gravity may be achieved *without modification of the Einstein-Maxwell equations,* through SUPERSYMMETRY, which eats up the strong gravitational field in the core of particle. Expelling gravity from the core of the KN spinning particle is similar to expelling the EM field from superconducting core, and both of these super-phenomena are realized in core of the KN solution by the supersymmertric Landau-Ginzburg field model [28, 29, 30, 31, 32, 33, 34, 21] in the form of a BPS-saturated Super-Bag solution, for which just the strong contradiction between Quantum and Gravity determines extreme sensitivity of the model to the choice of the separating surface **(R)**.

The natural choice of this surface for the KN solution was suggested by C. López [35]. According (1) and (2) it should be the "zero gravity" (ZG) surface

$$r = R = \frac{e^2}{2m}, \tag{5}$$

where function H vanishes

$$H_{KN}(R) = 0, \tag{6}$$

metric becomes flat, and can be matched with flat Minkowski space for $r < R$. It turns the López source of the KN solution in a shell-like bubble.

The corresponding metrics were suggested by Gürsay and Gürses [36]. They have the Kerr-Schild form (1) and retain the form of the Kerr congruence (4), while the function H is changed as follows,

$$H = \frac{f(r)}{r^2 + a^2 \cos^2\theta}, \tag{7}$$

where $f(r)$ is arbitrary smooth function of the Kerr radial coordinate r, taking at the far distances the KN form (2).

So far as in the Kerr solution r is oblate spheroidal coordinate [19], which is related with Cartesian coordinates by transformations

$$x + iy = (r + ia)\exp\{i\phi_K\}\sin\theta, \quad z = r\cos\theta, \quad \rho = r - t, \tag{8}$$

the bubble surface $r = R$ takes the oblate ellipsoidal form – the disk of the thickness R and radius $r_c = \sqrt{R^2 + a^2}$, where $a = J/m$.

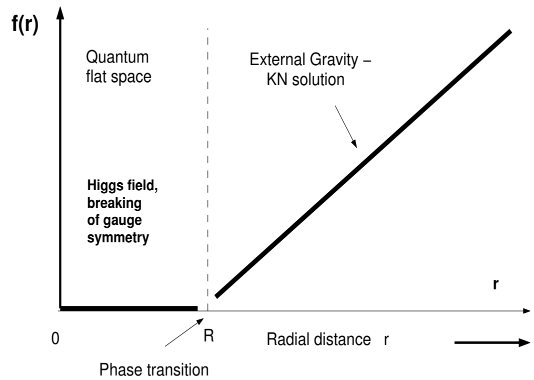

Figure 2: Behavior of the function $f(r)$ in bag model. Domain wall placed at $r = R$ separates flat Quantum interior from external KN Gravity .

For solutions without rotation, $a = 0$, and bubble turns into a sphere of the classical radius r_e. Such spherical shape was suggested by Dirac in [37] as an "extensible electron model" – prototype of the bag models, displaying one of their basic features of the bags – their deformability.

We see that deformations of the KN Super-Bag appear as consequence of the requirement on sharp separation of the zones **(I)**,**(E)**,**(R)**.

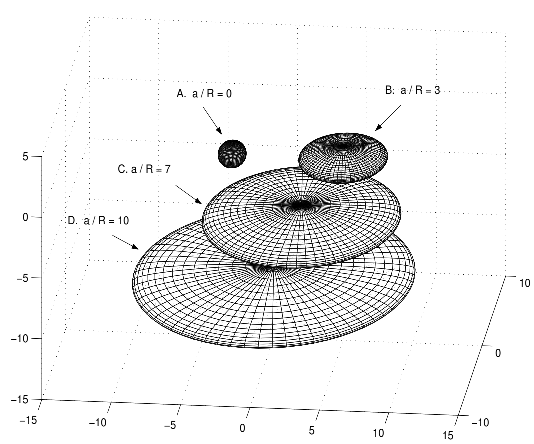

Figure 3: (A): Spherical bag without rotation $a/R = 0$, and disk-like bags for different values of the rotation parameter: (B)- $a/R = 3$; (C) - $a/R = 7$; and (D) - $a/R = 10$.

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3. Supersymmetry ensures consistency with gravity

3.1. Generalized Landau-Ginzburg field model and domain wall (DW) phase transition

The Landau-Ginzburg field model of superconductivity is used in many solitonic models, in particular, in the Nielsen-Olesen dual string model, as a field model in the MIT and SLAC bag models, and really, it is also the the Higgs model of symmetry breaking, because the Higgs vacuum itself "... is analog to a superconducting metal", [9]. The Landau-Ginzburg Lagrangian used in the Nielsen-Olesen model (minimal Landau-Ginzburg model) is

$$\mathcal{L}_{NO} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\mathcal{D}_\mu\Phi)(\mathcal{D}^\mu\Phi)^* - V(|\Phi|), \tag{9}$$

where $\mathcal{D}_\mu = \nabla_\mu + ieA_\mu$ are the $U(1)$ covariant derivatives, and $F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}$ is the corresponding field strength, and potential V has the quartic form

$$V = \lambda(\Phi^\dagger\Phi - \eta^2)^2, \tag{10}$$

where η is condensate of the Higgs field Φ, its vacuum expectation value (vev) $\eta < |\Phi| >$, [43].

The minimal Landau-Ginzburg model can be used to describe superconductivity inside the bag – interplay of the KN vector-potential with the Higgs condensate. Since requirements **(I),(E),(R)** define inside the bag a flat space, the corresponding covariant derivatives can be taken as flat,

$$\mathcal{D}_\mu = \nabla_\mu + ieA_\mu \ \rightarrow \ \mathcal{D}_\mu = \partial_\mu + ieA_\mu. \tag{11}$$

However, the NO and KN models have opposite spacial configurations: the KN bag model should describe a superconducting disk surrounded by the long-range EM and gravitational field, while the NO model describes vortex of the EM field inside the superconducting Higgs condensate which breaks the external long-range EM and gravitational field. Note, that this is a typical drawback of the most of soliton models and, in particular, the usual bag models which are formed as a "cavity in superconductor" [9]. The reason of this disadvantage lies in the use of the potential (10).

The correct opposite configuration – condensation of the Higgs field inside the core – requires more complex scalar potential V formed from several complex fields $\Phi_i, \ i = 1, 2, 3$, [22]. Kinetic part of the corresponding generalized LG model differs from those of the minimal Landau-Ginzburg model (9) only by summation over the fields Φ_i,

$$\mathcal{L}_{GLGkin} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\sum_i(\mathcal{D}_{i\mu}\Phi_i)(\mathcal{D}_i^\mu\Phi_i)^*, \tag{12}$$

while the potential V is changed very essentially, and has to be formed by analogy with machinery of the $N = 1$ supersymmetric field theory [21] from a superpotential function $W(\Phi_i)$.[1] The scalar potential[2]

$$V(r) = \sum_i F_i F_i^* \tag{13}$$

is formed through derivatives of the function $W(\Phi_i)$,

$$F_i = \partial W / \partial \Phi_i \equiv \partial_i W, \tag{14}$$

where

$$W(\Phi_i, \bar{\Phi}_i) = Z(\Sigma\bar{\Sigma} - \eta^2) + (Z + \mu) H \bar{H}, \tag{15}$$

μ and η are real constants, and the special notations are introduced $(H, Z, \Sigma) \equiv (\Phi_1, \Phi_2, \Phi_3)$, to identify Φ_1 as the complex Higgs field

$$H = |H| e^{i\chi}, \tag{16}$$

which interacts with the KN vector field A_μ as $\mathcal{D}_{1\mu} = \nabla_{1\mu} + ieA_\mu$. The fields Φ_2 and Φ_3 are assumed uncharged, and $\mathcal{D}_{i\mu} = \nabla_{i\mu}$ for $i = 2, 3$.

The condition $F_i = \partial_i W = 0$ determines two vacuum states with $V = 0$:

(I) internal vacua: $r < R - \delta$, where the Higgs field $|H| = \eta$, and $Z = -\mu, \ \Sigma = 0$,

and

(E) external vacuum state: $r > R + \delta$, where the Higgs field $H = 0$, and $Z = 0, \ \Sigma = \eta$,

separated by spike of the potential $V > 0$ in zone

(R) – a domain wall, interpolating between zones **(I)** and **(E)**, in the full correspondence with the requirements **(I),(E),(R)**.

Reduction of the corresponding LG equations to Bogomolny form is performed by minimization of the energy density per unit area of the DW surface,

$$\mu = \frac{1}{2} \sum_{i=1}^{3} \ [\sum_{\mu=0}^{3} |\mathcal{D}_\mu^{(i)} \Phi_i|^2 + |\partial_i W|^2]. \tag{17}$$

The four dimensional domain wall solutions in supersymmetric Landau-Ginzburg model have paid attention in the works [28, 29, 30, 31, 32, 33, 34], where it was usually considered the static planar domain walls positioned in (x,y) plane, with the transverse to the wall z-direction. However, even in the simplest case of the one field $\Phi(z)$ and one coordinate z,

$$\mu = \frac{1}{2}(|\partial_z \Phi|^2 + |\partial_\Phi W|^2), \tag{18}$$

[1] It is really not only analogy, and as we shell see, only one step differs this model from the true supersymmetric Higgs model, which was obtained by Morris in [48] with the purpose to get supersymmetric generalization of the Witten superconducting string model [49]. This model was used in [22, 23] to describe superconducting core of the KN solution.

[2] The signs bar $^-$ and star * both are used for complex conjugation.

reduction of the LG equation to Bogomolny form turns out to be nontrivial, since it requires the introduction of an arbitrary phase factor α, so that (18) can be equivalently presented in the form

$$\mu = \frac{1}{2}|\partial_z \Phi - e^{i\alpha}\partial_{\bar{\Phi}}\bar{W}|^2 + Re\ e^{i\alpha}\partial_z W, \tag{19}$$

which is saturated by the Bogomolny equation

$$\partial_z \Phi = e^{i\alpha}\partial_{\bar{\Phi}}\bar{W}. \tag{20}$$

The domain wall forming the KN bag is much more complicated, since first of all it is not planar, but forms the spheroidal boundary profile of which is shown in Fig.2. Second, it is formed by three chiral fields Φ_i, and thirdly, the most important feature is that this domain wall is not static and has non-trivial dependence on the phases of the complex fields Φ_i. The corresponding BPS saturated solution was found in [15, 24], where it was shown that the phases α_i of the complex fields Φ_i should acquire nontrivial dependence from time and angular coordinate

$$\alpha_1 = 2\chi(t,\phi), \quad \alpha_2 = \alpha_3 = 0, \tag{21}$$

and the Higgs field becomes oscillating, showing that just in the KN bag model the transformation to Bogomolny form (19) begins to operate at full power.

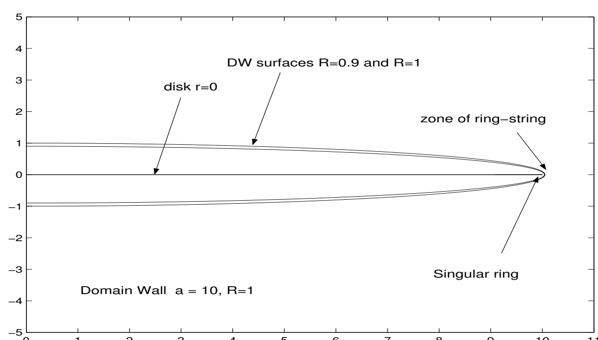

Figure 4: The domain wall profile (axial section) defined by the oblate spheroidal coordinate $r = R$.

3.2. Minimal Landau-Ginzburg model and quantization of the angular momentum

The non-trivial dependence (21) is fixed in zone **(I)**, where the generalized Landau-Ginzburg model is reduced to minimal LG model, and the Lagrangian (9) leads to equations

$$\Box A_\mu = J_\mu = e|H|^2(\chi,_\mu + eA_\mu). \tag{22}$$

One sees that vector potential A_μ acquires from the Higgs field the mass term $m_v = e|H|$, and the EM field becomes short-range, with the characteristic parameter $\lambda = 1/(e|H|)$ corresponding to the penetration depth of the EM field in superconductivity. As a consequence, the currents vanish inside the core, $J_\mu = 0$, leading to the equations

$$\Box A_\mu = 0, \quad \chi,_\mu + eA_\mu = 0, \tag{23}$$

showing that besides of the massive component $A_\mu^{m_v}$ which falls off receiving the mass m_v from the Higgs field, there are also the components of different behavior.

Vector-potential of the external KN solution (3) is

$$A_\mu dx^\mu = -\frac{er}{r^2 + a^2\cos^2\theta}(dr - dt - a\sin^2\theta d\phi). \tag{24}$$

It grows near the core and takes maximal value at the boundary of the disk, at $r = R = e^2/2m, \ \cos\theta = 0$,

$$A_\mu^{max} dx^\mu = -\ \frac{2m}{e}(dr - dt - ad\phi). \tag{25}$$

Note, that the component A_r is a perfect differential (as it is shown for example in [19]) and can be ignored. At the boundary, A_μ^{max} is dragged by the light-like direction of the Kerr singular ring (see Fig.3) and the component A_ϕ^{max} forms the closed Wilson loop, so that

$$e\oint A_\phi^{max} d\phi = 4\pi ma. \tag{26}$$

The right equation in (23) shows that penetrating inside the disk vector potential determines oscillating phase of the Higgs field as $\chi = 2mt + 2am\phi$. The condition of multiplicity of the periods χ and ϕ gives $2am = n, \ n = 1, 2, 3, ..$, which in view of $J = ma$, leads to quantization of angular momentum as

$$J = n/2, \ n = 1, 2, 3, ... \tag{27}$$

On the other hand (23) shows that phase of Higgs field

$$H = \Phi_1 = |H|e^{i(2mt+2am\phi)} \tag{28}$$

oscillates with the frequency $\omega = 2m$ which supports extension of the components $A_t^{in} = \frac{2m}{e}, \quad A_\phi^{in} = \frac{2ma}{e}$ inside the disk.[3] At the disk boundary (23) is broken, and according (22) there appear the surface currents J_μ.

[3]Note, that the left massless equation (23) is also satisfied, since $\Box A_t^{in} = 0$ is satisfied trivially. Also, $\Box A_\phi^{in} = 0$, because phase ϕ is analytic function of $(x + iy)$, leading to $\Box A_\phi^{in} = \partial\bar{\partial} A_\phi^{in} = 0$. These fields do not produce the field strength.

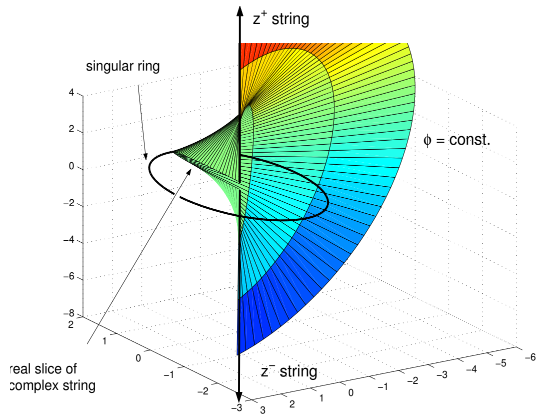

Figure 5: Kerr's coordinate $\phi = const$. Kerr singular ring drags the vector potential, forming a closed Wilson loop along edge border of the DW.

4. SuperBag as nonperturbative solution of the SuperQED model

4.1. Bosonic sector of the supersymmetric LG model

As we noticed earlier, the generalized Lndau-Ginzburg model based on the superpotential (15) is not true supersymmetric model. The difference is that the true superpotential W is to be a chiral function of the chiral superfields Φ_i, while the scalar potential

$$V = F_i F_i^* \tag{29}$$

is formed from the chiral part

$$F_i^* = \partial W / \partial \Phi_i, \tag{30}$$

but also incudes the antichiral superpotential $W^+(\Phi_i^+)$ depending on the antichiral superfields Φ_i^+

$$F_i = \partial W^+ / \partial \Phi_i^+. \tag{31}$$

These relations are retained in the bosonic sector of the supersymmetric theory, where the fields Φ_i and Φ_i^+ turn into the complex conjugate scalar components of the superfields.

To get full correspondence with supersymmetric theory, the fields Φ_i and $\bar{\Phi}_i$ in (15), should be considered as independent chiral fields Φ_i and $\tilde{\Phi}_i$, and there should also be introduced an antichiral superpotential $W^+(\Phi_i^+, \tilde{\Phi}_i^+)$, which in the bosonic sector turns into complex conjugated superpotential, built of the complex conjugated fields $\bar{W}(\Phi_i^*, \tilde{\Phi}_i^*)$. From the complex point of view, the transition from (15) to supersymmeric Higgs model may be considered as *complexification* of the moduli space – analytical extension from the real section, fixed by condition $\bar{\Phi}_i = \Phi_i^*$, to its complex extension, the manifold with independent coordinates Φ_i and $\tilde{\Phi}_i$, supplemented with complex conjugate coordinates Φ_i^*, $\tilde{\Phi}_i^*$. Therefore, the transition to bosonic sector of the supersymmetric generalized LG model requires doubling of the chiral field to eliminate their degeneracy on the real slice.

Returning to the original work by Morris [48], where the potential (15) was suggested for super-generalization of the Witten's superconducting string model [49], we should double the charged chiral fields Σ and Φ, and consider five chiral superfields $\Sigma_\pm, \Phi_\pm$, and Z, which in Witten's interpretation of this model as the $U(I) \times U'(I)$ Higgs field model, acquire the charges $(\pm 1, 0)$ for $\Phi_\pm$, and charges for the $\Sigma_\pm$ fields as $(0, \pm 1)$. The chiral superpotential (15) takes the form

$$W(\Phi_i, \tilde{\Phi}_i) = Z(\Sigma_+\Sigma_- - \eta^2) + (Z + \mu)\Phi_+\Phi_-, \tag{32}$$

with identification

$$\Phi_i = (\Phi_+, \Phi_-, \Sigma_+, \Sigma_-, Z). \tag{33}$$

The auxiliary fields

$$F_i^* = \partial W / \partial \Phi_i = (F_+^*, F_-^*, F_{\Sigma+}^*, F_{\Sigma-}^*, F_Z^*) \tag{34}$$

take the form

$$F_\pm^* = (Z + \mu)\Phi_\mp, \tag{35}$$

$$F_{\Sigma\pm}^* = Z\Sigma_\mp, \tag{36}$$

$$F_Z^* = \Sigma_+\Sigma_- + \Phi_+\Phi_- - \eta^2, \tag{37}$$

Vacuum expectation values of fields Φ_i for which $F_i^* = 0$ give minima of the potential $V = 0$ corresponding to supersymmetric vacuum states. Just as in case (15), we obtain two isolated vacua

(I) $\Phi_-\Phi_+ = \eta^2, \quad Z = -\mu, \quad \Sigma_+ = \Sigma_- = 0;$

(E) $\Phi_- = \Phi_+ = 0, \; Z = 0, \; \Sigma_+\Sigma_- = \eta^2;$

separated by the zone

(R) of the positive potential

$$\begin{aligned} V = &|\Sigma_+\Sigma_- + \Phi_+\Phi_- - \eta^2|^2 + |(Z + \mu)\Phi_+|^2 \\ &+ |(Z + \mu)\Phi_-|^2 + |Z|^2(|\Sigma_+|^2 + |\Sigma_-|^2). \end{aligned} \tag{38}$$

4.2. Transition to SuperQED model

We note that two oppositely charged superfields Φ_+ and Φ_- give rise to correspondence of the supersymmetric Landau-Ginzburg model to kinetic part of the Wess-Zumino SuperQED model [21],

$$\mathcal{L}_{SQEDkin} = -\frac{1}{4} W^a W_a + \Phi_+^+ e^{eV} \Phi_+|_{\theta\theta\bar{\theta}\bar{\theta}} + \Phi_-^+ e^{-eV} \Phi_-|_{\theta\theta\bar{\theta}\bar{\theta}}, \tag{39}$$

where V is vector superfield, and $W^a = -\frac{1}{4}\bar{D}\bar{D}D_\alpha V$. In the same time, the potential part (32) corresponds to the most general renormalizable supersymmetric Lagrangian and gives rise to nonperturbative generalization of the SuperQED model.

The chiral superfields $\Phi_\pm$, are expressed in the component form

$$\Phi_\pm(y) = H_\pm(y^\mu) + \sqrt{2}\theta\psi_\pm(y^\mu) + \theta\theta F_\pm(y^\mu), \tag{40}$$

as functions of the chiral coordinates $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ and θ, and the scalar components $H_\pm$ are *independent* Higgs fields, splitting of the complex conjugated Higgs field of the minimal Landau-Ginzburg model in (15) and (16). Interplay of the oppositely charged Higgs fields $H_\pm$ with vector potential in zone **(I)** is defined by (23) and yields

$$H_\pm = |H_\pm|e^{\pm i\chi}, \;\; \bar{H}_\pm = |H_\pm|e^{\mp i\chi}, \;\; \chi = 2mt + 2am\phi, \tag{41}$$

where the fields $\bar{H}_\pm$ are scalar components of the antichiral fields

$$\Phi^+_\pm(y^+) = \bar{H}_\pm(y^{+\mu}) + \sqrt{2}\bar{\theta}\bar{\psi}_\pm(y^{+\mu}) + \bar{\theta}\bar{\theta}\bar{F}_\pm(y^{+\mu}), \tag{42}$$

as functions of the antichiral coordinates $y^{+\mu} = x^\mu - i\theta\sigma^\mu\bar{\theta}$ and $\bar{\theta}$. The corresponding nonperturbative solution with doubled Higgs fields (41) can be obtained similar to [15].

In the Wess-Zumino SuperQED model, the two Weyl spinors $\psi^\pm$ in (40) combine into one massive Dirac spinor of the electron – superpartner of the Higgs doublet $H_\pm$, [21].

The nonperturbative super-bag solution generates in the core of spinning particle the flat Compton zone **(I)**, which is free from gravity and supersymmetric, representing the conditions for the work of the perturbative SuperQED model, while the remarkable perturbative properties of the WZ SuperQED model – "miracleous cancellations" of the component super-graphs [21] – form a link to perturbative QED. Note, that in the nonperturbative model of super-bag, the superpartners cannot be considered as separate particles, and are integrated as the superfield components of a single nonperturbative solution. The super-bag model reveals correspondence not only with gravity and electroweak sector of the SM, but also with a nonperturbative version of the SuperQED model.

5. Stringy structure

Usually, it is assumed that bags are deformed by rotations taking the shape of a string-like flux-tube joining the quark-antiquark pair [7].

In the KN Super-Bag, the spinning gravitational field controls disk-like shape of the bag, and string-like structure is formed for $a/R > 0$, at edge rim of the disk, as shown in Fig.4. In the equatorial plane, this string approaches very close to the Kerr singular ring, see Fig.4A, so, it is really just the singular ring regularized by the bag boundary.

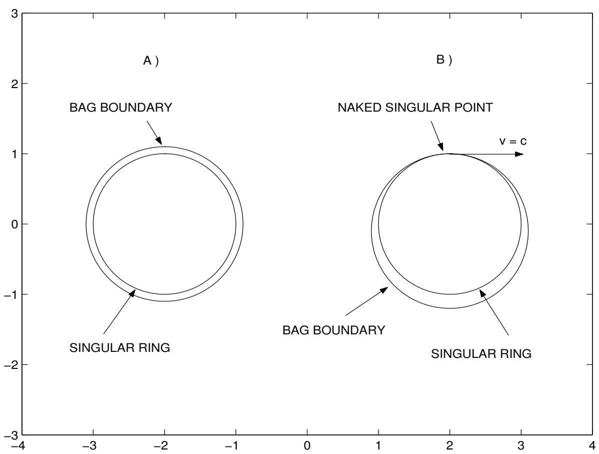

Figure 6: Regularization of the KN string. Boundary of bag fixes cut-off $R = r_e$ for the Kerr singular ring. A) The exact KN solution. B) The KN solution is excited by the lowest traveling mode: emergence of the singular pole.

Among diverse attempts to use nonperturbative models in the electroweak sector of the Standard Model (SM) [38, 39, 40, 41, 42], the central place takes the Nielsen-Olesen (NO) model [43, 44] of the string, which is created as a vortex line in a superconductor.

The assumption, that Kerr singular ring is similar to Nielsen-Olesen model of dual string was done very long ago in [46, 45], where it was noted that excitations of the KN solution create traveling waves along the Kerr ring. Later, it was obtained in [5, 6] close connection of the Kerr singular ring with the Sen fundamental string solution to low energy string theory.[4] In the KN bag model this string is formed at the sharp boundary of the superconducting disk, as a dual analog of the NO vortex line in superconductor.

In accordance with the condition (6), the KN gravity controls position of the bag boundary **(R)**, and also more thin effects, such as excitations of the KN gravity define dynamics of the bag and appearance of the traveling waves.

In particular, it has been shown [15], that the lowest EM excitation of the KN solution creates the traveling wave which has a circulating lightlike node. At this point, surface of the deformed bag touches the Kerr singular ring, as it is shown in Fig.4B, which breaks regularization at this point and creates the lightlike singular pole, which can be considered as emergence of the bare Dirac particle circulating inside the Compton zone of dressed

[4]Note also the complex N=2 critical string which was obtained in the complex Kerr geometry [47].

electron. On the other hand, this pole breaks homogeneity of the closed circular string, creating the frontal and rear ends turning this string in the open. As usual, the end points of an open string are associated with quarks, and the KN super-bag model turns into a single “bag-string-quark” system, 4D analog of D2-D1-D0-brane system of the string–M-theory.

6. Conclusions

We have considered principal features of the Kerr-Schild geometry which specify the supersymmetric bag model as a new way to particle physics consistent with gravity and electroweak sector of the SM. Two of these features are principally new, relative to the widespread belief:

– the spinning KN gravity is not weak, and becomes very strong at the Compton scale of the particle physics,

– compatibility between Quantum and Gravity can be achieved by means of supersymmetric generalization of the matter sector, without modification of the Einstein-Maxwell theory.

We considered interplay of the KN gravity with the matter sector based on the supersymmetric generalized LG field model, which is equivalent to supersymmetric Higgs mechanism of symmetry breaking, and give a nonperturbative solution to generalized Landau-Ginzburg field model in the form of a super-bag – nonperturbative version of the SuperQED model. By conception, the 4d super-bag model has to be soft and oscillating, similar to the conception of the superstring models [8, 25, 26].

Due to extreme high spin/mass ratio, impact of the gravitational KN field on the structure of space-time becomes very strong, and the consistent supersymmetric nonperturbative solutions become very sensible to the external Einstein-Maxwell field. As a result,

a) the super-bag model creates a free from gravity Compton core of spinning particle, where the supersymmetric vacuum state of the Higgs field provides the flat space, required for consistent work of quantum theory;

b) the super-bag takes the shape of a strongly oblate disk forming a circular string along its border;

c) gravitational and electromagnetic excitations of the KN solution create consistent stringy oscillations of the super-bag in the form of traveling waves.

Many problems remain to be solved. The closest is the so far unsolved problem of the exact nonstationary (oscillating or accelerating) generalization of the KN solution, the problem of the consistent solutions of the Dirac equation corresponding to confinement of quark inside the bag, and so on.

Nevertheless, the considered here features of the super-bag model are so intriguing that we risk to state that they really give the key to solution of the problem of unification of gravity with particle physics.

Finally, very important new aspect of this study is the direct link to non-perturbative Wess-Zumino SuperQED model, which provides remarkable cancellations between component diagrams, presenting a link between the nonperturbative bag-like solution and the conventional technics of the perturbative QED.

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