

Algebraic structures in exceptional geometry

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ABSTRACT

Exceptional field theory (EFT) gives a geometric underpinning of the U-duality symmetries of M-theory. In this talk I give an overview of the surprisingly rich algebraic structures which naturally appear in the context of EFT. This includes Borcherds superalgebras, Cartan type superalgebras (tensor hierarchy algebras) and L_∞ algebras. This is the written version of a talk based mainly on refs. [52, 61, 55, 76, 62, 78], presented at ISQS25, Prague, June 2017, at QTS-10/LT-12, Varna, June 2017, at SQS 2017, Dubna, Aug. 2017, and at $M \cap \Phi 9$, Belgrade, Sept. 2017.

Duality symmetries in string theory/M-theory mix gravitational and non-gravitational fields. Manifestation of such symmetries calls for a generalisation of the concept of geometry. It has been proposed that the compactifying space (torus) is enlarged to accommodate momenta (representing momenta and brane windings) in modules of a duality group. This leads to *double geometry* [1, 2, 4, 64, 6, 5, 8, 9, 10, 12, 11, 13, 14, 15, 45, 44, 28, 29, 60, 17, 18, 40, 41, 72, 73] in the context of T-duality, and *exceptional geometry* [7, 19, 20, 21, 22, 24, 25, 26, 27, 30, 31, 32, 33, 70, 34, 35, 56, 57, 49, 69, 71] in the context of U-duality. These classes of models are special cases of *extended geometries*, and can be treated in a unified manner [76]. The duality group is in a certain sense present already in the uncompactified theory. It becomes “geometrised”.

In the present talk, I will

- Describe the basics of extended geometry, with focus on the gauge transformations;
- Describe the appearance of Borcherds superalgebras and Cartan-type superalgebras (tensor hierarchy superalgebras);
- Indicate why L_∞ algebras provide a good framework for describing the gauge symmetries.

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- Point out some questions and directions.

The focus will thus be on algebraic aspects, and less on geometric ones.

Consider compactification from 11 to $11 - n$ dimensions on T^n . As is well known, fields and charges fall into modules of $E_{n(n)}$.

n	$E_{n(n)}$	R_1
3	$SL(3) \times SL(2)$	(3, 2)
4	$SL(5)$	10
5	$Spin(5, 5)$	16
6	$E_{6(6)}$	27
7	$E_{7(7)}$	56
8	$E_{8(8)}$	248
9	$E_{9(9)}$	fund

Table 1: A list of U-duality groups.

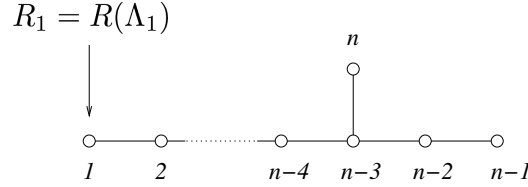


Figure 1: The module R_1 .

To be explicit, take $n = 7$ as an example. The gauge parameters ξ^M in **56** of E_7 decompose as:

$$\xi^m_7 + \lambda_{mn}_{21} + \tilde{\lambda}_{mnpqr}_{21} + \tilde{\xi}_{m,n_1\dots n_7}_7 \leftarrow \xi^M_{56} \quad (1)$$

We recognise the parameters for diffeomorphisms, gauge transformations of the 3-form and dual 6-form and a parameter for “dual diffeomorphisms”. The scalar fields are in the coset $E_{7(7)}/K(E_{7(7)}) = E_{7(7)}/(SU(8)/\mathbb{Z}_2)$. The dimension of coset is: $133 - 63 = 70$, and it is parametrised by

$$g_{mn}_{28} + C_{mnp}_{35} + \tilde{C}_{mnpqrs}_7 = \leftarrow G_{MN}_{70} \quad (2)$$

From the point of view of $N = 8$ supergravity in $D = 4$, this is the scalar field coset. Now it becomes a generalised metric. There are also mixed fields (generalised graviphotons): 1-forms in R_1 , etc.

The situation for T-duality is simpler. Compactification from 10 to 10 – d dimensions gives the (continuous) T-duality group $O(d, d)$. The momenta are complemented with string windings to form the $2d$ -dimensional module

Note that the continuous duality group is not to be seen as a global symmetry. Discrete duality transformations in $O(d, d; \mathbb{Z})$ or $E_{n(n)}(\mathbb{Z})$ arise as symmetries in certain backgrounds, roughly as the mapping class group $SL(n; \mathbb{Z})$ arises as discrete isometries of a torus. The rôle of the continuous versions of the duality groups is analogous to that of $GL(n)$ in ordinary geometry (gravity).

One has to decide how tensors transform. The generic recipe is to mimic the Lie derivative for ordinary diffeomorphisms:

$$L_U V^m = U^n \partial_n V^m - \partial_n U^m V^n . \quad (3)$$

The first term is a transport term, and the second one a \mathfrak{gl} transformation, with parameter in red.

In the case of U-duality, the role of GL is assumed by $E_{n(n)} \times \mathbb{R}^+$, and

$$\begin{aligned} \mathcal{L}_U V^M &= L_U V^M + Y^{MN} \partial_{PQ} U^P V^Q \\ &= U^N \partial_N V^M + Z^{MN} \partial_{PQ} U^P V^Q , \end{aligned} \quad (4)$$

where $Z^{MN} \partial_{PQ} = -n P_{\text{adj}}^M \partial_Q^N + \partial_N M_Q \partial_P = Y^{MN} \partial_{PQ} - M_P \partial_N Q$ projects on the adjoint of $E_{n(n)} \times \mathbb{R}$, so that the red factor becomes a parameter for an $\mathfrak{e}_n \oplus \mathbb{R}$ transformation.

The transformations form an “algebra” for $n \leq 7$:

$$[\mathcal{L}_U, \mathcal{L}_V] W^M = \mathcal{L}_{[U, V]} W^M , \quad (5)$$

where the “Courant bracket” is $[U, V]^M = \frac{1}{2}(\mathcal{L}_U V^M - \mathcal{L}_V U^M)$, provided that the derivatives fulfil a “*section constraint*”.

The section constraint ensures that fields locally depend only on an n -dimensional subspace of the coordinates, on which a $GL(n)$ subgroup acts. It reads $Y^{MN} \partial_{PQ} \dots N = 0$, or

$$(\otimes)|_{\overline{\mathbb{R}}_2} = 0 . \quad (6)$$

For $n \geq 8$ more local transformations, so called “ancillary transformations” [76] emerge, which are constrained local transformations in \mathfrak{g} .

n	R_1	R_2
3	$(\mathbf{3}, \mathbf{2})$	$(\bar{\mathbf{3}}, \mathbf{1})$
4	$\mathbf{10}$	$\bar{\mathbf{5}}$
5	$\mathbf{16}$	$\mathbf{10}$
6	$\mathbf{27}$	$\bar{\mathbf{27}}$
7	$\mathbf{56}$	$\mathbf{133}$
8	$\mathbf{248}$	$\mathbf{1} \oplus \mathbf{3875}$

Table 2: A list of R_1 and R_2 for different E_n .

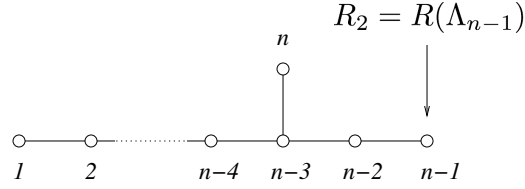


Figure 2: The module R_2 .

The interpretation of the section condition is that the momenta locally are chosen so that they may span a linear subspace of cotangent space with maximal dimension, such that any pair of covectors p, p' in the subspace fulfil $(p \otimes p')|_{\bar{R}_2} = 0$.

The corresponding statement for double geometry is $\eta^{MN}{}_M \otimes_N = 0$, where η is the $O(d, d)$ -invariant metric. The maximal linear subspace is a d -dimensional isotropic subspace, and it is determined by a pure spinor Λ . Once a Λ is chosen, the section condition can be written $\Gamma^M \Lambda_M = 0$. An analogous linear construction can be performed in the exceptional setting. The section condition in double geometry derives from the level matching condition in string theory. Locally, supergravity is recovered. Globally, non-geometric solutions are also obtained.

There is a universal form [52, 55, 76] of the generalised diffeomorphisms for any Kac–Moody algebra and choice of coordinate representation. Let the coordinate representation be $R(\lambda)$, for λ a fundamental weight dual to a simple root α (the construction can be made more general). Then

$$\sigma Y = -\eta_{AB} T^A \otimes T^B + (\lambda, \lambda) + \sigma - 1, \quad (7)$$

where η is the Killing metric and σ the permutation operator, $\sigma(a \otimes b) = (b \otimes a)\sigma$.

This follows from the existence of a solution to the section constraint in the form of a linear space:

- Each momentum must be in the minimal orbit. Equivalently, $p \otimes p \in \overline{R(2\lambda)}$.
- Products of different momenta may contain $\overline{R(2\lambda)}$ and $\overline{R(2\lambda - \alpha)}$, where $R(2\lambda - \alpha)$ is the highest representation in the antisymmetric product. Expressing these conditions in terms of the quadratic Casimir gives the form of Y .

I will skip the detailed description of the generalised gravity. It effectively provides the local dynamics of gravity and 3-form, which are encoded by a vielbein E_M^A in the coset $(E_{n(n)} \times \mathbb{R})/K(E_{n(n)})$.

n	$E_{n(n)}$	$K(E_{n(n)})$
3	$SL(3) \times SL(2)$	$SO(3) \times SO(2)$
4	$SL(5)$	$SO(5)$
5	$Spin(5, 5)$	$(Spin(5) \times Spin(5))/\mathbb{Z}_2$
6	$E_{6(6)}$	$USp(8)/\mathbb{Z}_2$
7	$E_{7(7)}$	$SU(8)/\mathbb{Z}_2$
8	$E_{8(8)}$	$Spin(16)/\mathbb{Z}_2$
9	$E_{9(9)}$	$K(E_{9(9)})$

Table 3: A list of compact subgroups.

The T-duality case is described by a generalised metric in the coset $O(d, d)/(O(d) \times O(d))$, parametrised by the ordinary metric and B -field.

With some differences from ordinary geometry, one can go through the construction of connection, torsion, metric compatibility etc., and arrive at generalised Einstein's equations encoding the equations of motion for all fields. (This has been done for $n \leq 8$.)

For $n \geq 8$, the coset $E_{n(n)}/K(E_{n(n)})$ contains higher mixed tensors that do not carry independent physical degrees of freedom. They are removed by ancillary transformations that arise in the commutator between generalised diffeomorphisms [70, 56, 57, 55, 76].

One may introduce (local) supersymmetry. In the case of T-duality, the superspace is based on the fundamental representation of an orthosymplectic supergroup $OSp(d, d|2s)$. The exceptional cases are unexplored, but will be based on ∞ -dimensional superalgebras [50].

The generalised diffeomorphisms do not satisfy a Jacobi identity. On general grounds, it can be shown that the ‘‘Jacobiator’’

$$[[U, V], W] + \text{cycl} \neq 0, \quad (8)$$

but is proportional to $([U, V], W) + \text{cycl}$, where $(U, V) = \frac{1}{2}(\mathcal{L}_U V + \mathcal{L}_V U)$.

It is important to show that the Jacobiator in some sense is trivial. It turns out that $\mathcal{L}_{(U,V)}W = 0$ (for $n \leq 7$), and the interpretation is that it is a gauge transformation with a parameter representing reducibility (for $n \leq 6$). (The limits on n in the statements here are due to non-covariance of the derivative arising at some point in the tensor hierarchy, see below. I will not go into details.)

In double geometry, this reducibility is just the scalar reducibility of a gauge transformation: $\delta B_2 = d\lambda_1$, with the reducibility $\delta\lambda_1 = d\lambda'_0$.

In exceptional geometry, the reducibility turns out to be more complicated, leading to an infinite (but well defined) reducibility, containing the modules of tensor hierarchies, and providing a natural generalisation of forms (having connection-free covariant derivatives).

The reducibility continues, and there are ghosts at all levels > 0 . The representations are those of a “tensor hierarchy”, the sequence of representations R_n of n -form gauge fields in the dimensionally reduces theory.

$$R_1 \xleftarrow{\partial} R_2 \xleftarrow{\partial} R_3 \xleftarrow{\partial} \dots \tag{9}$$

Example, $n = 5$:

$$\mathbf{16} \xleftarrow{\partial} \mathbf{10} \xleftarrow{\partial} \overline{\mathbf{16}} \xleftarrow{\partial} \mathbf{45} \xleftarrow{\partial} \overline{\mathbf{144}} \xleftarrow{\partial} \dots \tag{10}$$

$$16 - 10 + 16 - 45 + 144 - \dots = 11 \tag{11}$$

(suitably regularised), which is the number of degrees of freedom of a pure spinor.

The representations $\{R_n\}_{n=1}^\infty$ agree with [51]

- The ghosts for a “pure spinor” constraint (a constraint implying an object lies in the minimal orbit);
- The positive levels of a Borchers superalgebra $\mathcal{B}(E_n)$.

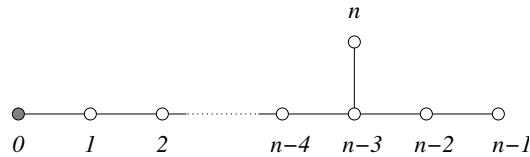


Figure 3: Dynkin diagram for $\mathcal{B}(E_n)$.

Indeed, the denominator appearing in the denominator formula for $\mathcal{B}(E_n)$ is identical to the partition function of a “pure spinor” [51].

$$\begin{aligned} \mathcal{B}(D_n) &\approx \mathfrak{osp}(n, n|2) \\ \mathcal{B}(A_n) &\approx \mathfrak{sl}(n+1|1) \end{aligned}$$

$$\dots \xleftarrow{\partial} R_{-1} \xleftarrow{\partial} R_0 \xleftarrow{\partial} \underbrace{R_1 \xleftarrow{\partial} R_2 \xleftarrow{\partial} \dots \xleftarrow{\partial} R_{8-n}}_{\text{covariant}} \xleftarrow{\partial} R_{9-n} \xleftarrow{\partial} R_{10-n} \xleftarrow{\partial} \dots \tag{12}$$

The modules R_1, \dots, R_{8-n} behave like forms. The “exterior derivative” is connection-free (for a torsion-free connection), and there is a wedge product [32].

The modules show a symmetry: $R_{9-n} = \overline{R}_n$. There is another extension to negative levels that respects this symmetry, and seems more connected to geometry: tensor hierarchy algebras [61, 62]

In the classification of finite-dimensional superalgebras by Kac, there is a special class, “Cartan-type superalgebras”. The Cartan-type superalgebra $W(n)$, which I prefer to call $W(A_{n-1})$, is asymmetric between positive and negative levels, and (therefore) not defined through generators corresponding to simple roots and Serre relations.

$W(A_{n-1})$ is the superalgebra of derivations on the superalgebra of (point-wise) forms in n dimensions.

Any operation $\omega \rightarrow \Omega \wedge \iota_V \omega$, where Ω is a form and V a vector, belongs to $W(A_{n-1})$. A basis is given by

level = 1	ι_a
0	$e^b \iota_a$
-1	$e^{b_1} e^{b_2} \iota_a$
-2	$e^{b_1} e^{b_2} e^{b_3} \iota_a$
...	...

The level decomposition of $W(A_{n-1})$.

A subalgebra $S(A_{n-1})$ contains traceless tensors. The positive levels agree with $\mathcal{B}(A_{n-1}) \approx \mathfrak{sl}(n|1)$. Note that the representations of torsion and torsion Bianchi identity appear at levels -1 and -2 .

In spite of the absence of a Cartan involution, there is a way to give a systematic Chevalley–Serre presentation of the superalgebra, based on the same Dynkin diagram as the Borchers superalgebra [62].

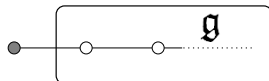


Figure 4: Dynkin diagram for $\mathcal{B}(\mathfrak{g})$ and $W(\mathfrak{g})$.

The construction can be extended to $W(D_n)$, and, most interestingly, $W(E_n)$ (and the corresponding $S(\mathfrak{g})$). The statements about torsion and Bianchi identities remain true (but we still lack a good geometric argument).

Back to the Jacobi identity. Expressed in terms of a fermionic ghost in R_1 ,

$$[[c, c], c] \neq 0. \quad (13)$$

How is this remedied? The most general formalism for gauge symmetries is the Batalin–Vilkovisky formalism, where everything is encoded in the master equation $(S, S) = 0$.

If transformations are field-independent, one may consider the ghost action consistently. An L_∞ algebra is a (super)algebraic structure which provides a perturbative solution to the master equation.

Let C denote *all* ghosts. Then the master equation states the nilpotency of a transformation

$$\delta C = (S, C) = C + [C, C] + [C, C, C] + [C, C, C, C] + \dots \quad (14)$$

The identities that need to be fulfilled are:

$$\begin{aligned} {}^2C &= 0, \\ [C, C] + 2[C, C, C] &= 0, \\ [C, C, C] + 2[[C, C], C] + 3[C, C, C, C] &, \\ \dots & \end{aligned} \quad (15)$$

Assuming $c = 0$, the non-vanishing of $[[c, c], c]$ can be compensated by the derivative of an element in R_2 (representing reducibility). One needs to introduce a 3-bracket

$$[c, c, c] \in R_2. \quad (16)$$

Then, there are more identities to check.

For double field theory, a 3-bracket is enough [77].

For exceptional field theory, there are signs, that one will in fact obtain arbitrarily high brackets [78]. There are also other issues concerning the non-covariance outside the “form window”. I will not go into detail.

In conclusion, the area has rich connections to various areas of pure mathematics, some of which are under investigation:

- Group theory and representation theory
- Minimal orbits
- Superalgebras
- Cartan-type superalgebras
- Infinite-dimensional (affine, hyperbolic,...) Lie algebras
- Geometry and generalised geometry
- Automorphic forms
- L_∞ algebras
- ...

There are many open questions:

- Can the formalism be continued to $n > 9$? The transformations work for *e.g.* E_{10} [76], and there seems to be no reason (other than mathematical difficulties) that it stops there. Is there a connection to the “ E_{10} proposal” [79] with emergent space?

- *Geometry from algebra?* What is the precise geometric relation between the tensor hierarchy algebra and the generalised diffeomorphisms?
- *Superspace/supergeometry?* And some formalism generalising that of pure spinor superfields, that manifests supersymmetry?
- *The section constraint:* Can it be lifted, or dynamically generated?
- What can be learnt about the full string theory/M-theory?
- ... ?

Thank you for your attention.

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