# Non-Riemannian generalizations of Born-Infeld models and trace free gravitational equations*

**Diego Julio Cirilo-Lombardo**†
CONICET- Universidad de Buenos Aires.
Instituto de Fisica del Plasma (INFIP).
Buenos Aires, Argentina, Universidad de Buenos Aires.
Facultad de Ciencias Exactas y Naturales.
Departamento de Fisica. Buenos Aires, Argentina
and
Bogoliubov Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research,
141980 Dubna, Russian Federation

Abstract

Non-Riemannian generalization of the standard Born-Infeld (BI) Lagrangian is reviewded in this talk from a theory of gravitation with dynamical torsion field. The field equations derived from the proposed action lead to a trace free gravitational equation (non-riemannian analog to the trace free equation (TFE) from[1][2][3]) and the field equations for the torsion respectively. In this theoretical context, the fundamental constants arise all from the same geometry through geometrical invariant quantities (as from the curvature R). New results involving generation of primordial magnetic fields and the link with leptogenesis and baryogenesis are presented and possible explanations given. The physically admisible matter fields can be introduced in the model via the torsion vector h . Such fields include some dark matter candidates such as axion, right neutrinos and Majorana and moreover, physical observables as vorticity can be included in the same way. From a new wormhole soluton in a cosmological spacetime with torsion we also show that the primordial cosmic magnetic fields can originate from h with the axion field (that is con tained in h ) the responsible to control the dynamics and stability of the cosmic magnetic field but not the magnetogenesis itself. The analisys of Grand Unified Theories (GUT) in the context of this model indicates that the group manifold candidates are based in SO(10), SU(5) or some exceptional groups as E(6),E (7), etc. Hints about astrophysical consequences of this formulation are given

* This work has been supported by *CONICET Argentina*
† e-mail address: diego777jcl@gmail.com

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## 1. Introduction

In this talk we resume the results of previous works [1][2][3][4] and references therein. The idea to construct a complete geometrization of the physics is very old. The drawback of the Einstein GR (General Relativity) equations is the RHS: $R_{\alpha\beta} - \frac{g_{\alpha\beta}}{2}R = \kappa T_{\alpha\beta}$ with the symmetric tensor (non-geometrical) $\kappa T_{\alpha\beta}$ that introduces *heuristically* the energy-momentum distribution. Similar drawbacks are contained by the unimodular gravity. It is well known that the unimodular gravity is obtained from Einstein-Hilbert action in which the *unimodular condition:* $\sqrt{-\det g_{\mu\nu}} = 1$ is also imposed from the very beginning The resulting field equations correspond to the *traceless* Einstein equations and can be shown that they are equivalent to the full Einstein equations with the cosmological constant term $\Lambda$, where $\Lambda$ enters as an integration constant and the equivalence between unimodular gravity and general relativity is given by the arbitrary value of lamda. On the other hand the idea that the cosmological term arises as an integration constant is one of the motivations for the study of the unimodular gravity and also in the context of supergravity. The fact that the determinant of the metric is fixed has clearly profound consequences on the structure of given theory. First of all, it reduces the full group of diffeomorphisms to invariance under the group of unimodular general coordinate transformations which are transformations that leave the determinant of the metric unchanged.

Similar thing happens in the non-Riemannian case, as pointed out in[**?**][**?**][**?**][**?**][**?**], where the corresponding affine geometrical structure induces naturally the following constraint: $\frac{K}{g} = constant$. This natural constraint impose a condition (ratio) between both basic tensors through their determinants: the metric determinant $g$ and the fundamental one $K$ (in the sense of a nonsymmetric theory that contains the antisymmetric structures), independently of the precise functional form of $K$ or $g$. In this work our starting point will be precisely the last one, where a *metric affine structure* in the space-time manifold (as described in Section II) will be considered. We will also show that trace free gravitational equations can be naturally obtained when the Lagrangian function (geometrical action) is taken as a measure involving a particular combination of the fundamental tensors of the geometry:

$$\sqrt{|\det f\left(g_{\mu\nu}, f_{\mu\nu}, R_{\mu\nu}\right)|}$$

with the (0,2) tensors $g_{\mu\nu}, f_{\mu\nu}, R_{\mu\nu}$ : the symmetric metric, the antisymmetric one(that acts as potential of the torsion field) and the generalized Ricci tensor (proper of the non Riemannian geometry). The three tensors are related with a Clifford structure of the tangent space (for details see[4]) where the explicit choice for $f\left(g_{\mu\nu}, f_{\mu\nu,}R_{\mu\nu}\right)$ is given in Section III. This type of Lagrangians, because are non-Riemannian generalizations of the well known Nambu-Goto and Born-Infeld (BI) ones, can be physically and geometrically analyzed. Due the pure geometrical structure of the theory, induced energy momentum tensors and fundamental constants (actually functions) *emerge* naturally. Consequently, this fact allows the physical realization of the Mach principle that is briefly treated in SectionVIII after the (trace free) dynamic equations in Section IV are obtained.

In Section V the trace free gravitational equations and the meaning of the cosmological term as integration constant are discussed from the physical point of view, meanwhile in Section VI the constancy of G (Newton constant) is similarly discussed. The important role played by the dual of the torsion field as geometrical energy-momentum tensor is given in Section VII. Some physical consequences of the model, as the geometrical origin of the $\alpha\Omega-$dynamo, is presented in Section IX that it is very important because establish the link between the mathematical structure of the model of the first part of the article and the physics of the early universe and the particle physics of the second half of this work. In Section X the direct relation between the torsion with axion electrodynamics and Chern-Simons (CS) theory is discussed considering the geometrical structure of the dual vector of the torsion field. In Section XI an explanation about the magnetogenesis in FRW scenario, the structure of the GUT where the SM is derived and the role of the axion in the dynamics of the cosmic magnetic field is presented. Finally some concluding remarks are given in Section XII.

## 2. Basis of the metrical-affine geometry

The starting point is a hypercomplex construction of the (metric compatible) spacetime manifold

$$M,\, g_{\mu\nu} \equiv e_\mu \cdot e_\nu \tag{1}$$

where for each point $p \in M$ there exists a local affine space $A$. The connection over $A$, $\widetilde{\Gamma}$, define a generalized affine connection $\Gamma$ on $M$, specified by $(\nabla, K)$, where $K$ is an invertible $(1,1)$ tensor over $M$. We will demand for the connection to be compatible and rectilinear, that is

$$\nabla K = KT, \qquad \nabla g = 0 \tag{2}$$

where $T$ is the torsion, and $g$ the space-time metric (used to raise and lower the indices and determining the geodesics), that is preserved under parallel transport. This generalized compatibility condition ensures that the generalized affine connection $\Gamma$ maps autoparallels of $\Gamma$ on $M$ into straight lines over the affine space $A$ (locally). The first equation above is equal to the condition determining the connection in terms of the fundamental field in the $UFT$ non-symmetric. Hence, $K$ can be identified with the fundamental tensor in the non-symmetric fundamental theory. This fact gives us the possibility to restrict the connection to a (anti-)Hermitian theory.

The covariant derivative of a vector with respect to the generalized affine connection is given by

$$A^{\mu}{}_{;\nu} \equiv A^{\mu}{}_{,\nu} + \Gamma^{\mu}{}_{\alpha\nu} A^{\alpha} \tag{3}$$

$$A_{\mu;\nu} \equiv A_{\mu}{}_{,\nu} - \Gamma^{\alpha}{}_{\mu\nu} A_{\alpha} \tag{4}$$

The generalized compatibility condition (2) determines the 64 components of the connection by the 64 equations

$$K^{\mu}_{\nu;\alpha} = K^{\mu}_{\rho} T^{\rho}{}_{\nu\alpha} \quad \text{where} \quad T^{\rho}{}_{\nu\alpha} \equiv 2\Gamma^{\rho}{}_{[\alpha\nu]} \tag{5}$$

Notice that by contracting indices $\nu$ and $\alpha$ in the first equation above, an additional condition over this hypothetical fundamental (nonsymmetric) tensor $K$ is obtained

$$K_{\mu\alpha;}{}^{\alpha} = 0$$

that, geometrically speaking, reads

$$d^{*}K = 0.$$

This is a current-free condition over the tensor $K$. Notice that the metric is used here to down the indices (metric compatible space-time) and consequently we can work also with $K_{\alpha\nu} = g_{\alpha\beta} K^{\beta}_{\nu}$

The metric is uniquely determined by the metricity condition, which puts 40 restrictions on the derivatives of the metric

$$g_{\mu\nu,\rho} = 2\Gamma_{(\mu\nu)\rho} \tag{6}$$

The space-time curvature tensor, that is defined in the usual way, has two possible contractions: the Ricci tensor $R^{\lambda}_{\mu\lambda\nu} = R_{\mu\nu}$, and the second contraction $R^{\lambda}_{\lambda\mu\nu} = 2\Gamma^{\lambda}{}_{\lambda[\nu,\mu]}$, which is identically zero due to the metricity condition (2).

In order to find a symmetry of the torsion tensor, let us denote the inverse of $K$ by $\widehat{K}$. Therefore, $\widehat{K}$ is uniquely specified by condition $\widehat{K}^{\alpha\rho}$ $K_{\alpha\sigma} = K^{\alpha\rho}\widehat{K}_{\alpha\sigma} = \delta^{\rho}_{\sigma}$.

As it was pointed out in , inserting explicitly the torsion tensor as the antisymmetric part of the connection in (5), and multiplying by $\frac{1}{2}\widehat{K}^{\alpha\nu}$, results, after straighforward computations, in

$$\left(Ln\sqrt{-K}\right)_{,\mu} - \Gamma^{\nu}_{(\mu\nu)} = 0 \tag{7}$$

where $K = \det\,(K_{\mu\rho})$. Notice that from expression (7) we arrive at the relation between the determinants $K$ and $g$:

$$\frac{K}{g} = \text{constant}$$

(strictly a constant scalar function of the coordinates). Now we can write

$$\Gamma^{\nu}{}_{\alpha\nu,\beta} - \Gamma^{\nu}{}_{\beta\nu,\alpha} = \Gamma^{\nu}{}_{\nu\beta,\alpha} - \Gamma^{\nu}{}_{\nu\alpha,\beta}\,, \tag{8}$$

as the first term of (7) is the derivative of a scalar. Then, the torsion tensor has the symmetry

$$T^{\nu}{}_{\nu[\beta,\alpha]} = T^{\nu}{}_{\nu[\alpha,\beta]} = 0 \tag{9}$$

This implies that the trace of the torsion tensor, defined as $T^{\nu}{}_{\nu\alpha}$, is the gradient of a scalar field

$$T_{\alpha} = \nabla_{\alpha}\phi \tag{10}$$

Expressions precisely as (1) and (2) ensure that the basic non-symmetric field structures (*i.e.* $K$) take on a definite geometrical meaning when interpreted in terms of affine geometry. Notice that the tensor K carries the 2-form (bivector) that will be associated with the fundamental antisymmetric form in the next Sections. Such antisymmetric form is introduced from the tangent space via the generalization of the Ambrose-Singer theorem by exponentiation.

## 3. Geometrical Lagrangians: the generalized Born-Infeld action

Let us start with the geometrical Lagrangian introduced in [[**?**][**?**][**?**][**?**][**?**]]

$$\mathcal{L}_g = \sqrt{\det\left[\lambda\left(g_{\alpha\beta} + F_{\alpha\beta}\right) + R_{\alpha\beta}\right]} \tag{11}$$

it can be rewritten as

$$\mathcal{L}_g = \sqrt{\det\left(G_{\alpha\beta} + \mathcal{F}_{\alpha\beta}\right)} \tag{12}$$

with the following redefinitions

$$G_{\alpha\beta} = \lambda g_{\alpha\beta} + R_{(\alpha\beta)} \quad \text{and} \quad \mathcal{F}_{\alpha\beta} = \lambda F_{\alpha\beta} + R_{[\alpha\beta]} \tag{13}$$

where a totally antisymmetric torsion tensor $T^{\alpha}_{\gamma\beta} = \varepsilon^{\alpha}_{\gamma\beta\delta}h^{\delta}$ is assumed($h^{\delta}$ its dual vector field). Notice that the antisymmetric tensor $F_{\alpha\beta}$,that takes the role of the electromagnetic field, is proportional to the dual of the potential for the (totally antisymmetric) torsion field. A brief review on the origin of this type of Lagrangians in the context of unified theories in reductive geometries is in Appendix I of [1]. Consequently the generalized Ricci tensor splits into a symmetric and antisymmetric part, namely:

$$R_{\mu\nu} = \overbrace{\overset{\circ}{R}_{\mu\nu} - T_{\mu\rho}{}^{\alpha}T_{\alpha\nu}{}^{\rho}}^{R_{(\mu\nu)}} + \overbrace{\overset{\circ}{\nabla}_{\alpha}T_{\mu\nu}{}^{\alpha}}^{R_{[\mu\nu]}}$$

where $\overset{\circ}{R}_{\mu\nu}$ is the general relativistic Ricci tensor constructed with the Christoffel connection. The expansion of the determinant leads to the Born-Infeld generalization in the usual form :

$$\mathcal{L}_g = \sqrt{|G|}\sqrt{1 + \frac{1}{2}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{16}\left(\mathcal{F}_{\mu\nu}\widetilde{\mathcal{F}}^{\mu\nu}\right)^2} \tag{14}$$

$$= \Lambda^2\sqrt{|g|}\sqrt{1 + \frac{1}{2}\Lambda_1^2 F_{\mu\nu}F^{\mu\nu} - \frac{1}{16b^4}\left(\Lambda_2^2 F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2} \tag{15}$$

where

$$\Lambda = \lambda + \frac{g_{\alpha\beta}R^{(\alpha\beta)}}{4} \tag{16}$$

$$\Lambda_1^2 = \lambda^2\left(1 + \frac{2}{\lambda}\frac{F_{\mu\nu}R^{[\mu\nu]}}{F_{\mu\nu}F^{\mu\nu}} + \frac{1}{\lambda^2}\frac{R_{[\mu\nu]}R^{[\mu\nu]}}{F_{\mu\nu}F^{\mu\nu}}\right) \tag{17}$$

$$\Lambda_2^2 = \lambda^2\left(1 + \frac{2}{\lambda}\frac{F_{\mu\nu}\widetilde{R}^{[\mu\nu]}}{F_{\mu\nu}\widetilde{F}^{\mu\nu}} + \frac{1}{\lambda^2}\frac{R_{[\mu\nu]}\widetilde{R}^{[\mu\nu]}}{F_{\mu\nu}\widetilde{F}^{\mu\nu}}\right) \tag{18}$$

Although the action is exact and have the correct limit, the analysis can be simplest and substantially improved using the following action

$$\mathcal{L}_{gs} = \sqrt{\det\left[\lambda g_{\alpha\beta}\left(1 + \frac{R_s}{4\lambda}\right) + \lambda F_{\alpha\beta}\left(1 + \frac{R_A}{\lambda}\right)\right]} \tag{19}$$

$$R_s \equiv g^{\alpha\beta}R_{(\alpha\beta)}; \qquad R_A \equiv f^{\alpha\beta}R_{[\alpha\beta]} \tag{20}$$

$\left(with\ f^{\alpha\beta} \equiv \frac{\partial \ln(\det F_{\mu\nu})}{\partial F_{\alpha\beta}}, \det F_{\mu\nu} = 2F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)$ that contains all necessary information and is more suitable to manage. If the induced structure from the tangent space $T_p(M)$(via Ambrose-Singer theorem) is intrinsically related to a (super)manifold structure, we have seen that there exists a particular transformation where the details are given in [1][2]

## 4. Field equations

The geometry of the space-time Manifold is to be determined by the Noether symmetries

$$\frac{\delta L_G}{\delta g^{\mu\nu}} = 0, \qquad \frac{\delta L_G}{\delta f^{\mu\nu}} = 0 \tag{21}$$

where the functional (Hamiltonian) derivatives in the sense of Palatini (in this case with respect to the potentials), are understood. The choice "measure-like" form for the geometrical Lagrangian $L_G$ (reminiscent of a nonlinear sigma model), as is evident, satisfy the following principles:

*i) the principle of the natural extension of the Lagrangian density as square root of the fundamental line element containing also $F_{\mu\nu}$.*

*ii) the symmetry principle between $g_{\mu\nu}$ and $F_{\mu\nu}$(e.g. $g_{\mu\nu}$ and $F_{\mu\nu}$should enter into $L_G$ symmetrically)*

*iii)the principle that the spinor symmetry, namely*

$$\nabla_\mu g_{\lambda\nu} = 0, \qquad \nabla_\mu \sigma_{\lambda\nu} = 0 \tag{22,23}$$

*with*

$$g_{\lambda\nu} = \gamma_\lambda \cdot \gamma_\nu, \qquad \sigma_{\lambda\nu} = \gamma_\lambda \wedge \gamma_\nu \sim *F_{\lambda\nu} \tag{24,25}$$

*should be derivable from $L_G$(21)*

The last principle is key because it states that the spinor invariance of the fundamental space-time structure should be derivable from the dynamic symmetries given by (21). The fact that the $L_G$ satisfies the 3 principles shows also that it has the simpler form.

Notice that the action density proposed by Einstein in in his nonsymmetric field theory satisfies i) and ii) but not iii).

**Remark 1** *Due the totally antisymmetric character of the torsion field it is completely determined by the fundamental (structural 2-form) antisymmetric tensor, and consequently the variations must acquire the form given by expression (21)**: metric and torsion have each one their respective potentials that are in coincidence with the fundamental structure of the geometry.***

### 4.1. $\delta_g L_G$

The starting point for the metrical variational procedure is in the same way as in the standard Born-Infeld theory: from the following factorization of the geometrical Lagrangian :

$$\mathcal{L} = \sqrt{|g|}\sqrt{\det(\alpha\lambda)}\sqrt{1 + \frac{1}{2b^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{16b^4}\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2} \equiv \sqrt{|g|}\sqrt{\det(\alpha\lambda)}\mathbb{R} \tag{26}$$

where

$$b = \frac{\alpha}{\beta} = \frac{1 + (R_S/4\lambda)}{1 + (R_A/4\lambda)}, \qquad R_S = g^{\alpha\beta}R_{\alpha\beta}, \qquad R_A = f^{\alpha\beta}R_{\alpha\beta}, \tag{27,28,29}$$

and $\lambda$ an arbitrary constant we perform the variational metric procedure with the following result (details see Appendix II)

$$\delta_g \mathcal{L} = 0 \Rightarrow R_{(\alpha\beta)} - \frac{g_{\alpha\beta}}{4} R_s = \frac{R_s}{2\mathbb{R}^2\alpha^2}\left[F_{\alpha\lambda}F_{\beta}{}^{\lambda} - F_{\mu\nu}F^{\mu\nu}\frac{R_{(\alpha\beta)}}{R_s}\right] + \tag{30, 31}$$

$$+\frac{R_s}{4\mathbb{R}^2\alpha^2 b^2}\left[F_{\mu\nu}\widetilde{F}^{\mu\nu}\left(\frac{F_{\eta\rho}\widetilde{F}^{\eta\rho}}{8}g_{\alpha\beta} - F_{\alpha\lambda}\widetilde{F}_{\beta}{}^{\lambda}\right) + \frac{F_{\eta\rho}\widetilde{F}^{\eta\rho}}{2}\frac{R_{(\alpha\beta)}}{R_s}\right] +$$

$$+2\lambda\left[g_{\alpha\beta} + \frac{1}{\mathbb{R}^2\alpha^2}\left(F_{\alpha\lambda}F_{\beta}{}^{\lambda} + \frac{F_{\mu\nu}\widetilde{F}^{\mu\nu}}{2b^2}\left(\frac{F_{\eta\rho}\widetilde{F}^{\eta\rho}}{8}g_{\alpha\beta} - F_{\alpha\lambda}\widetilde{F}_{\beta}{}^{\lambda}\right)\right)\right],$$

**Remark 2** *Notice that:*

*1) The eq. (31) is* trace-free *type, consequently the trace of the third term of the above equation ( that is the cosmological one ) is equal to zero. This happens trivially if $\lambda = 0$ or $4\mathbb{R}^2\alpha^2 = -\left(F_{\alpha\lambda}F^{\ \alpha\lambda} - \frac{\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2}{4b^2}\right)$ In terms of the Maxwell Lagrangian we have $(\mathbb{R}\alpha)^2 = \left(L_{Maxwell} + \frac{\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2}{16b^2}\right) \equiv \mathcal{W}\left(I_S, I_P, b\right)$ that allow us to simplify the eq. (31) once more as follows*

$$R_{(\alpha\beta)} - \frac{g_{\alpha\beta}}{4} R_s = \frac{R_s}{2\mathcal{W}}\left[F_{\alpha\lambda}F_{\beta}{}^{\lambda} - F_{\mu\nu}F^{\mu\nu}\frac{R_{(\alpha\beta)}}{R_s}\right] +$$

$$+\frac{R_s}{4\mathcal{W}b^2}\left[F_{\mu\nu}\widetilde{F}^{\mu\nu}\left(\frac{F_{\eta\rho}\widetilde{F}^{\eta\rho}}{8}g_{\alpha\beta} - F_{\alpha\lambda}\widetilde{F}_{\beta}{}^{\lambda}\right) + \frac{F_{\eta\rho}\widetilde{F}^{\eta\rho}}{2}\frac{R_{(\alpha\beta)}}{R_s}\right] +$$

$$+2\lambda\left[g_{\alpha\beta} + \frac{1}{\mathcal{W}}\left(F_{\alpha\lambda}F_{\beta}{}^{\lambda} + \frac{F_{\mu\nu}\widetilde{F}^{\mu\nu}}{2b^2}\left(\frac{F_{\eta\rho}\widetilde{F}^{\eta\rho}}{8}g_{\alpha\beta} - F_{\alpha\lambda}\widetilde{F}_{\beta}{}^{\lambda}\right)\right)\right],$$

*2) b takes the place of limiting parameter (maximum value) for the electromagnetic field strength.*

*3)b is not a constant in general, in sharp contrast with the Born-Infeld or string theory cases.*

*4) Because b is the ratio $\frac{\alpha}{\beta} = \frac{1+(R_S/4\lambda)}{1+(R_A/\lambda 4)}$ involving both curvature scalars from the contractions of the generalized Ricci tensor: it is preponderant when the symmetrical contraction of $R_{\alpha\beta}$ is greater than the skew one.*

*5) The fact pointed out in ii), namely that the curvature scalar plays the role as some limiting parameter of the field strength, was conjectured by Mansouri in [**?**] in the context of gravity theory over group manifold (generally with symmetry breaking). In such a case, this limit was established after the explicit integration of the internal group-valuated variables that is not our case here.*

*6) In similar form that the Eddington conjecture:* $R_{(\alpha\beta)} \propto g_{\alpha\beta}$*, we have a condition over the ratios as follows:*

$$\frac{R_{(\alpha\beta)}}{R_s} \propto \frac{g_{\alpha\beta}}{D} \tag{32}$$

*that seems to be universal.*

*7)The equations are the simplest ones when* $b^{-2} = 0$ $(\beta = 0)$*,taking the exact "quasilinear" form*

$$R_{(\alpha\beta)} - \frac{g_{\alpha\beta}}{4}R_s = \underbrace{\frac{R_s}{2\alpha^2}\left[F_{\alpha\lambda}F_{\beta}{}^{\lambda} - F_{\mu\nu}F^{\mu\nu}\frac{R_{(\alpha\beta)}}{R_s}\right]}_{Maxwell-like} + 2\lambda\underbrace{\left[g_{\alpha\beta} + \frac{1}{\mathcal{W}}F_{\alpha\lambda}F_{\beta}{}^{\lambda}\right]}_{\widetilde{g}_{eff}}, \tag{33}$$

*this particular case (e.g. projective invariant) will be used through this work. Notice that when* $b^{-2} = 0$ $(\beta = 0)$ *all terms into the gravitational equation (31) involving the pseudoscalar invariant, namely* $F_{\mu\nu}\widetilde{F}^{\mu\nu}$ *or* $F_{\alpha\lambda}\widetilde{F}_{\beta}{}^{\lambda}$ *,vanishes. Consequently we arrive to the simplest expression (33) that will be used in Section XI for example.*

## 4.2. $\delta_f L_G$

Let us to take as starting point the geometrical Lagrangian (19)

$$\mathcal{L}_{gs} = \sqrt{\det\left[\lambda g_{\alpha\beta}\left(1 + \frac{R_s}{4\lambda}\right) + \lambda F_{\alpha\beta}\left(1 + \frac{R_A}{4\lambda}\right)\right]} \tag{34}$$

$$= \sqrt{|g|}\lambda^2\alpha^2\left(\sqrt{1 + \frac{1}{2}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{16}\left(\mathcal{F}_{\mu\nu}\widetilde{\mathcal{F}}^{\mu\nu}\right)^2}\right) \tag{35}$$

then, having into account that : $R_A = f^{\mu\nu}R_{\mu\nu}$ and $\frac{\partial \ln(\det F_{\mu\nu})}{\partial F_{\alpha\beta}} = f^{\alpha\beta}$ (due that $b$ that contains $R_A$ must be also included in the variation)we obtain

$$\frac{\delta L_G}{\delta F_{\sigma\omega}} = 0 \rightarrow \left(\frac{\sqrt{|g|}\lambda\beta}{2\mathbb{R}b}\right)\left[\mathbb{F}^{\sigma\omega}\beta - \frac{\mathbb{F}}{4\lambda}R_{[\mu\nu]}\chi^{\mu\nu\sigma\omega}\right] = 0 \tag{36}$$

where: $\mathbb{F} \equiv \left[F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}b^{-2}\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2\right]$, $\mathbb{F}^{\sigma\alpha} \equiv \left[F^{\sigma\alpha} - \frac{1}{4}b^{-2}\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)\widetilde{F}^{\sigma\alpha}\right]$ and $\chi^{\mu\nu\sigma\omega} \equiv f^{\mu\omega}f^{\sigma\nu} - f^{\mu\sigma}f^{\omega\nu}$. Notice that the quantity $b = \alpha/\beta$ (concretely $\beta$) was also varied in the above expression given the second term in (36).

Contracting (36) with $F_{\alpha\beta}$, a condition over the curvature and the electromagnetic field invariants is obtained as

$$\left(\frac{\sqrt{|g|}\lambda\beta}{\mathbb{R}b}\right)\mathbb{F}\left[\beta - \frac{R_A}{2\lambda}\right] = 0$$

This condition is satisfied for $R_A = -4\lambda$ is the exact projective invariant case (that correspond with $\beta = 0$), and for $R_A = 2\lambda$.

**Remark 3** *the variational equation (36) is a dynamic equation for the torsion field in complete analogy with the eqs. (31) for the curvature.*

## 5. Emergent trace free gravitational equations: the meaning of $\Lambda$

Starting from the trace free equation (31) *that is not assumed* but arises from the model, the task is to rewrite it as

$$\underbrace{\overset{\circ}{R}_{\alpha\beta} - \frac{g_{\alpha\beta}}{2}\overset{\circ}{R}}_{\equiv G_{\alpha\beta}} = \underbrace{6\left(-h_\alpha h_\beta + \frac{g_{\alpha\beta}}{2} h_\gamma h^\gamma\right)}_{\equiv T^h_{\alpha\beta}} + \frac{g_{\alpha\beta}}{2} R_s + T^F_{\alpha\beta} + 2\lambda\rho_{\alpha\beta} \qquad (37)$$

where

$$\rho_{\alpha\beta} \equiv g_{\alpha\beta} + \frac{1}{\mathcal{W}}\left(F_{\alpha\lambda}F_\beta{}^\lambda + \frac{F_{\mu\nu}\widetilde{F}^{\mu\nu}}{2b^2}\left(\frac{F_{\eta\rho}\widetilde{F}^{\eta\rho}}{8} g_{\alpha\beta} - F_{\alpha\lambda}\widetilde{F}_\beta{}^\lambda\right)\right) \qquad (38)$$

$$T^F_{\alpha\beta} \equiv \frac{R_s}{2\mathcal{W}}\left\{\left(F_{\alpha\lambda}F_\beta{}^\lambda - F_{\mu\nu}F^{\mu\nu}\frac{R_{(\alpha\beta)}}{R_s}\right) + \right. \qquad (39)$$

$$\left. + \frac{1}{2b^2}\left[F_{\mu\nu}\widetilde{F}^{\mu\nu}\left(\frac{F_{\eta\rho}\widetilde{F}^{\eta\rho}}{8} g_{\alpha\beta} - F_{\alpha\lambda}\widetilde{F}_\beta{}^\lambda\right) + \frac{\left(F_{\eta\rho}\widetilde{F}^{\eta\rho}\right)^2}{2}\frac{R_{(\alpha\beta)}}{R_s}\right]\right\}$$

the LHS of (37) is the Einstein tensor. The "GR" divergence $\overset{\circ}{\nabla}{}^\alpha$ of $G_{\alpha\beta}$is zero because is a geometrical geometrical identity and in an analog manner $\overset{\circ}{\nabla}{}^\alpha\left(T^h_{\alpha\beta} + T^F_{\alpha\beta}\right) = 0$ because both tensors have the same symmetry that the corresponding GR energy momentum tensors of a vector field and electromagnetic field respectively:

$$\overset{\circ}{\nabla}{}^\alpha G_{\alpha\beta} = \overset{\circ}{\nabla}{}^\alpha\left(T^h_{\alpha\beta} + T^F_{\alpha\beta}\right) = 0$$

consequently the remaining part must be a covariantly constant tensor that we *assume* proportional to $g_{\alpha\beta}$ :

$$\nabla^\alpha\left(\frac{g_{\alpha\beta}}{2} R_s + 2\lambda\rho_{\alpha\beta}\right) = 0$$

$$\Rightarrow \left(\frac{g_{\alpha\beta}}{2} R_s + 2\lambda\rho_{\alpha\beta}\right) = \Lambda g_{\alpha\beta} \to R_s = 2\Lambda \qquad (40)$$

Coming back to the original trace free expressions we have the expected formula

$$\underbrace{\overset{\circ}{R}_{\alpha\beta} - \frac{g_{\alpha\beta}}{2}\overset{\circ}{R}}_{\equiv G_{\alpha\beta}} = \underbrace{6\left(-h_\alpha h_\beta + \frac{g_{\alpha\beta}}{2}h_\gamma h^\gamma\right)}_{\equiv T^h_{\alpha\beta}} + T^F_{\alpha\beta} + \Lambda g_{\alpha\beta} \tag{41}$$

**Remark 4** *Tracing the first expression in (40) we have* $R_s = 2\Lambda = \overset{\circ}{R} + 6h_\mu h^\mu$ *linking the value of the curvature and the norm of the torsion vector field. Consequently, if the dual of the torsion field have the role of the energy-matter carrier, the meaning of lambda as the vacuum energy is immediately established.*

**Remark 5** *Notice that the LHS in expression (40) instead to be proportional to the metric tensor it can be proportional to the square of a Killing-Yano tensor.*

## 6. On the constancy of G

At this level, no assertion can state with respect to $G$ or even with respect to $c$. The link with the general relativistic case is given by the identification of electromagnetic energy-momentum tensor with the term analogous $T^F_{\alpha\beta}$ in our metric variational equations:

$$\frac{8\pi G}{c^4}\left(F_{\alpha\lambda}F_\beta{}^\lambda - F_{\mu\nu}F^{\mu\nu}\frac{g_{\alpha\beta}}{4}\right) \to \frac{R_s}{2\mathcal{W}}\left(F_{\alpha\lambda}F_\beta{}^\lambda - F_{\mu\nu}F^{\mu\nu}\frac{R_{(\alpha\beta)}}{R_s}\right)$$

Consequently we have:

$$\kappa = \frac{8\pi G}{c^4} \to \frac{R_s}{2\mathbb{R}^2\alpha^2}$$

$$\text{and} \quad \frac{g_{\alpha\beta}}{4} = \frac{R_{(\alpha\beta)}}{R_s}$$

The above expression indicates that the ratio must remains constant due the Noether symmetries and conservation laws of the field equations. Notice that (as in the case of $b$) there exist a limit for all the physical fields coming from the *geometrical invariants quantities.*

## 7. The vector $h_\mu$ and the energy-matter interpretation

One of the characteristics that more attract the attention in unified field theoretical models is the possibility to introduce the energy and matter through its geometrical structure. In our case the torsion field takes the role of RHS of the standard GR gravity equation by mean its dual, namely $h_{\mu}$.

Consequently, in order to explain the physical role of $h_{\mu}$, we know (due the Hodge-de Rham decomposition [Appendix III]) that it can be decomposed as:

$$h_{\alpha} = \nabla_{\alpha}\Omega + \varepsilon_{\alpha}^{\beta\gamma\delta}\nabla_{\beta}A_{\gamma\delta} + \gamma_1 \overbrace{\varepsilon_{\alpha}^{\beta\gamma\delta}M_{\beta\gamma\delta}}^{axial\ vector} + \gamma_2 \overbrace{P_{\alpha}}^{polar\ vector} \tag{42}$$

where $\gamma_1$and $\gamma_2$ can be phenomenologically related to physical constants (e.g: $\gamma_1 = \frac{8\pi}{c}\sqrt{G}$ is a physical constant related to the Blackett formula ). From the eq.motion for the torsion namely: $\nabla_{\alpha}T^{\alpha\beta\gamma} = -\lambda F^{\beta\gamma}$and coming back to (42) we obtain the following important equation

$$\overset{\circ}{\Box}A_{\gamma\delta} - \gamma\left[\nabla_{\alpha}M^{\alpha}{}_{\gamma\delta} + (\nabla_{\gamma}P_{\delta} - \nabla_{\delta}P_{\gamma})\right] = -\lambda F_{\gamma\delta} \tag{43}$$

Let us consider, in particular, the case when $\lambda F_{\gamma\delta} \rightarrow 0$ :

$$\overset{\circ}{\Box}A_{\gamma\delta} = \gamma\left[\nabla_{\alpha}M^{\alpha}{}_{\gamma\delta} + (\nabla_{\gamma}P_{\delta} - \nabla_{\delta}P_{\gamma})\right] \tag{44}$$

We can immediately see that, if $M^{\alpha}{}_{\gamma\delta}$ is identified with the intrinsic spin angular momentum of the ponderable matter, $P_{\delta}$ is its lineal momentum vector and $A_{\gamma\delta}$ is the gravitational radiation tensor, then eq.(44) states that the sum of the intrinsic spin angular momentum and the orbital angular momentum of ponderable matter is conserved if the gravitational radiation is absent., if $M^{\alpha}{}_{\gamma\delta}$ is identified with the intrinsic spin angular momentum of the ponderable matter, $P_{\delta}$ is its lineal momentum vector and $A_{\gamma\delta}$ is the gravitational radiation tensor, then eq.(44) states that the sum of the intrinsic spin angular momentum and the orbital angular momentum of ponderable matter is conserved if the gravitational radiation is absent.

### 7.1. Killing-Yano systems and the vector $h_{\mu}$

Without enter in many details (these will be treated somewhere) the antisymmetric tensor $A_{\gamma\delta}$ in the $h_{\beta}$ composition is related with the Killing and Killing-Yano systems. Consequently we can introduce two types of couplings into the $A_{\gamma\delta}$ divergence : it correspond with the generalized current interpretation that also has $h_{\mu}$.

i) Defining

$$A_{\gamma\delta} \equiv A_{[\gamma;\delta]} \tag{45}$$

such that

$$\overset{\circ}{\nabla}_{\rho}A_{[\gamma;\delta]} = \frac{4\pi}{3}\left(j_{[\gamma}\,g_{\delta]\rho}\right) \tag{46}$$

then , in this case we can identify $A_{\gamma\delta} = 2F_{\gamma\delta}$ because $F_{\gamma;\delta}^{\delta} = 4j_{\gamma}$ and $A_{[\gamma\delta;\rho]} = F_{[\gamma\delta;\rho]} = 0$

In this case the contribution of $A_{\gamma\delta}$ to $h_\beta$ is null.

ii) Let us consider now a fully antisymmetric coupling as

$$A_{[\gamma;\delta];\rho} = \frac{4\pi}{3} j_{[\gamma} F_{\delta]\rho} \tag{47}$$

, having into account the vorticity vector also

$$\omega_\mu \equiv u^\lambda \varepsilon_{\lambda\mu\nu\rho} \nabla^\nu u^\rho \tag{48}$$

and considering a plasma with electrons, protons etc.

$$j^\gamma \sim A^\gamma + q_s n_s u_s^\gamma \tag{49}$$

where $A_\mu$ is the vector potential and $q_s$ is the particle charge, $n_s$is the number density (in the rest frame) and the four-velocity of species s is $u_s^\gamma$. In this case $h_\alpha$takes the form

$$h_\alpha = \nabla_\alpha \Omega + \varepsilon_\alpha^{\beta\gamma\delta} \nabla_\beta A_{\gamma\delta} + \gamma_1 \varepsilon_\alpha^{\beta\gamma\delta} M_{\beta\gamma\delta} + \gamma_2 P_\alpha \rightarrow \tag{50}$$

$$h_\alpha = \nabla_\alpha \Omega + \varepsilon_\alpha^{\gamma\delta\rho} \frac{4\pi}{3} j_{[\gamma} F_{\delta]\rho} - \gamma_1 u^\lambda \varepsilon_{\lambda\alpha\nu\rho} \nabla^\nu u^\rho + \gamma_2 P_\alpha \tag{51}$$

$$h_\alpha = \nabla_\alpha \Omega + \varepsilon_\alpha^{\gamma\delta\rho} \frac{4\pi}{3} \left[A + q_s n_s u_s\right]_{[\gamma} F_{\delta]\rho} - \gamma_1 u^\lambda \varepsilon_{\lambda\alpha\nu\rho} \nabla^\nu u^\rho + \gamma_2 P_\alpha \tag{52}$$

Consequently in 3+1 decomposition we have (overbar correspond to spacial 3-dim. vectors)

$$h_0 = \nabla_0 \Omega + \frac{4\pi}{3} \overline{j} \cdot \overline{B} + \gamma_1 \overline{u} \cdot \left(\overline{\nabla} \times \overline{u}\right) + \gamma_2 P_0 \tag{53}$$

$$h_0 = \nabla_0 \Omega + \frac{4\pi}{3} \left[\overline{A} \cdot \left(\overline{\nabla} \times \overline{A}\right) + q_s n_s \overline{u}_s \cdot \overline{B}\right] + \gamma_1 \overline{u} \cdot \left(\overline{\nabla} \times \overline{u}\right) + \gamma_2 P_0 \tag{54}$$

and

$$h_i = \nabla_i \Omega + \frac{4\pi}{3} \left[-\left(\overline{j} \times \overline{E}\right)_i + j_0 \overline{B}_i\right] + \gamma_1 \left[u_0 \left(\overline{\nabla} \times \overline{u}\right) + \left(\overline{u} \times \overline{\nabla} u_0\right) + \left(\overline{u} \times \dot{\overline{u}}\right)\right]_i + \gamma_2 P_i \tag{55}$$

$$h_i = \nabla_i \Omega + \frac{4\pi}{3} \left[-\left(\left(\overline{A} + q_s n_s \overline{u}_s\right) \times \overline{E}\right)_i + \left(\Phi + q_s n_s u_{0s}\right) \overline{B}_i\right] + \tag{56}$$

$$+ \gamma_1 \left[u_0 \left(\overline{\nabla} \times \overline{u}\right) + \left(\overline{u} \times \overline{\nabla} u_0\right) + \left(\overline{u} \times \dot{\overline{u}}\right)\right]_i + \gamma_2 P_i$$

Notice that in $h_0$ we can recognize the magnetic and vortical helicities

$$h_0 = \nabla_0 \Omega + \frac{4\pi}{3} \left[h_M + q_s n_s \overline{u}_s \cdot \overline{B}\right] + \gamma_1 h_V + \gamma_2 P_0 \tag{57}$$

The above expression will be very important in the next sections, in particular to discuss magnetogenesis and particle generation. Notice the important fact that the symmetry of the vorticity can be associated to a 2-form bivector.

## 8. Physical consequences

In this Section we will make contact with the physical consequences of the model. Firstly we introduce the 3+1 splitting for axisymmetric space-times that is useful from the the physical viewpoint for the analysis of the electrodynamic equations with high degree of nonlinearity, as in our case. Secondly we take the 3+1 field equations in the in the linear limit where the induction equations (dynamo) are obtained, showing explicitly the important role of the torsion field as the generator of a purely geometric $\alpha$-term. Thirdly, we derive the geometrical analog of the Lorentz force and the elimination of the electric field from the induction equations. Also, the origin of the seed magnetic field via the geometrical $\alpha$-term generated by the torsion vector is worked out.

### 8.1. Electrodynamic structure in 3+1

The starting point will be the line element in $3+1$ splitting(Appendix IV of [1]): the 4-dimensional space-time is split into 3-dimensional space and 1-dimensional time to form a foliation of 3-dimensional spacelike hypersurfaces. The metric of the space-time is consequently, given by $ds^2 = -\alpha^2 dt^2 + \gamma_{ij}\left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right)$ where $\gamma_{ij}$ is the metric of the 3-dimensional hypersurface,$\alpha$ is the lapse function, and $\beta^i$ is the shift function (see Appendix IV of[1] for details) . For any nonlinear Lagrangian, in sharp contrast with the Einstein-Maxwell case, the field equations $d{*}\mathbb{F} = {*}\mathbb{J}$ and the Bianchi-geometrical condition $dF = 0$ (where we have defined the Hodge dual $*$ and $\mathbb{F} = \frac{\partial \mathcal{L}}{\partial F}$) are expressed by the vector fields

$$E, B, \mathbb{E} = \frac{\partial \mathcal{L}}{\partial E}, \mathbb{B} = \frac{\partial \mathcal{L}}{\partial B} \tag{58}$$

that live into the slice. In our case given by the geometrical Lagrangian $\mathcal{L}_g$ (not be confused with the Lie derivative$\mathcal{L}_\beta$!)

$$\nabla \cdot \mathbb{E} = -\overline{h} \cdot \mathbb{B} + 4\pi\rho_e \tag{59}$$

$$\nabla \cdot B = 0 \tag{60}$$

$$\begin{aligned} \nabla \times (\alpha E) &= -(\partial_t - \mathcal{L}_\beta) B \\ &= -\partial_0 B + (\beta \cdot \nabla) B - (B \cdot \nabla) \beta \end{aligned} \tag{61}$$

$$\begin{aligned} \nabla \times (\alpha \mathbb{B}) + h_0 \mathbb{B} - \overline{h} \times \mathbb{E} &= -(\partial_t - \mathcal{L}_\beta)\mathbb{E} + 4\pi\alpha j \\ &= \partial_0 \mathbb{E} - (\beta \cdot \nabla) \mathbb{E} + (\mathbb{E} \cdot \nabla) \beta + 4\pi\alpha j \end{aligned} \tag{62}$$

where $h^\mu$ is the torsion vector. Notice that, here and the subsequent Sections, the overbar indicates 3-dimensional space vectors.

### 8.2. Dynamo effect and geometrical origin of $\alpha\Omega$ term[3]

In the case of weak field approximation and $\left(F^{01} \to E^{i},\ F^{jk} \to B^{i}\right)$the electromagnetic Maxwell-type equations in 3+1 take the form

$$\nabla_{\nu} F^{\nu\mu} = T^{\mu\nu\rho} F_{\nu\rho} = \varepsilon^{\mu\nu\rho}{}_{\delta} h^{\delta} F_{\nu\rho} \qquad (d^{*}F = {}^{*}J) \tag{63}$$

$$\overline{\nabla} \cdot \overline{E} = -\overline{h} \cdot \overline{B} \tag{64}$$

$$\partial_t \overline{E} - \overline{\nabla} \times \overline{B} = h^0 \overline{B} - \overline{h} \times \overline{E} \tag{65}$$

and

$$\nabla_{\nu}{}^{*}F^{\nu\mu} = 0 \qquad (dF = 0) \tag{66}$$

$$\overline{\nabla} \cdot \overline{B} = 0 \tag{67}$$

$$\partial_t \overline{B} = -\overline{\nabla} \times \overline{E} \tag{68}$$

Putting all together, the set of equations is

$$\overline{\nabla} \cdot \overline{E} + \overline{h} \cdot \overline{B} = \rho_{ext} \tag{69}$$

$$\partial_t \overline{E} - \overline{\nabla} \times \overline{B} = h^0 \overline{B} - \overline{h} \times \overline{E} - \sigma_{ext} \left[\overline{E} + \overline{v} \times \overline{B}\right] \tag{70}$$

$$\overline{\nabla} \cdot \overline{B} = 0 \tag{71}$$

$$\partial_t \overline{B} = -\overline{\nabla} \times \overline{E} \tag{72}$$

where we have introduced external charge density and current. Following the standard procedure we take the rotational to the second equation above obtaining straightforwardly the modified dynamo equation

$$\overline{\nabla} \times \partial_t \overline{E} + \overline{\nabla}^2 \overline{B} = \overline{\nabla} \times \left(h^0 \overline{B}\right) + \left(\overline{h} \cdot \overline{B} - \rho_{ext}\right) \overline{h} + \left(\overline{\nabla} \cdot \overline{h}\right) \overline{E} - \sigma_{ext} \left[\partial_t \overline{B} + \left(\overline{\nabla} \cdot \overline{v}\right) \overline{B}\right] \tag{73}$$

where the standard identities of the vector calculus plus the first, the third and the fourth equations above have been introduced. Notice that in the case of the standard approximation and (in the spirit of this research) without any external or additional ingredients, we have

$$\overline{\nabla}^2 \overline{B} = h^0 \left(\overline{\nabla} \times \overline{B}\right) + \left(\overline{h} \cdot \overline{B}\right) \overline{h} + \left(\overline{\nabla} \cdot \overline{h}\right) \overline{E} \tag{74}$$

Here we can see that there exist and $\alpha$- term with a pure geometrical origin (and not only a turbulent one) that is given by $h^0$ (the zero component of the dual of the torsion tensor).

### 8.3. The generalized Lorentz force

An important point in any theory beyond relativity is the concept of force. As is known, general relativity has deficiencies at this point. Now we are going to show that it is possible to derive from our proposal the Lorentz force as follows. From expression (32) the geometrical induced current is recognized

$$\partial_t\overline{E} - \overline{\nabla} \times \overline{B} = h^0\overline{B} - \overline{h} \times \overline{E} \equiv \overline{J} \tag{77}$$

$$\overline{J} \times \overline{B} = \left(h^0\overline{B} - \overline{h} \times \overline{E} - \overline{j}_{ext}\right) \times \overline{B} \tag{78}$$

$$= -\left[\left(\overline{h} \cdot \overline{B}\right)\overline{E} - \left(\overline{E} \cdot \overline{B}\right)\overline{h}\right] - \overline{j}_{ext} \times \overline{B} \tag{79}$$

we assume $j_{ext}$ proportional to the velocity and other contributions. Consequently, reordering terms from above, a geometrically induced Lorentz-like force arises

$$\left(\overline{J} + \overline{j}_{ext}\right) \times \overline{B} = -\left[\underbrace{\left(\overline{h} \cdot \overline{B}\right)}_{\rho_{geom}}\overline{E} - \left(\overline{E} \cdot \overline{B}\right)\overline{h}\right] \rightarrow \tag{80}$$

$$\underbrace{\left(\overline{h} \cdot \overline{B}\right)}_{\rho_{geom}}\overline{E} + \underbrace{\left(\overline{J} + \overline{j}_{ext}\right)}_{j_{gen}} \times \overline{B} = \left(\overline{E} \cdot \overline{B}\right)\overline{h} \rightarrow \text{Lorentz induced force} \tag{81}$$

being the responsible of the induced force, the torsion vector itself. Notice, from the above equation, the following issues:

1)The external currents are identified with $\overline{J}$

2) We can eliminate the electric field in standard form

$$\overline{E} = \frac{\left(\overline{E} \cdot \overline{B}\right)\overline{h} - \overline{j}_{gen} \times \overline{B}}{\left(\overline{h} \cdot \overline{B}\right)} \tag{82}$$

being the above expression very important in order to replace the electric field into the dynamo equation, introducing naturally the external current in the model.

### 8.4. Generalized current and $\alpha$-term

In previous paragraph we have derived a geometrical induced Lorentz force where the link between the physical world and the proposed geometrical model is through a generalized current $j_{gen}$. An important fact of that expression is that it is possible to eliminate the electric field (and insert it into the equation of induction) as follows.

From the formula of the induction, namely

$$\overline{\nabla}^2\overline{B}+\underbrace{\overline{\nabla}\times\left(-h^0\overline{B}+\overline{h}\times\overline{E}\right)}_{\mathcal{E}_{Geom}}=0 \tag{83}$$

and using the eq. (49) to eliminate the electric field as function of the torsion, the generalized current and the magnetic field respectively:

$$\overline{h}\times\overline{E}=\frac{-\overline{h}\times\left(\overline{j}_{gen}\times\overline{B}\right)}{\left(\overline{h}\cdot\overline{B}\right)}=-\frac{\left(\overline{h}\cdot\overline{B}\right)\overline{j}_{gen}-\left(\overline{h}\cdot\overline{j}_{gen}\right)\overline{B}}{\left(\overline{h}\cdot\overline{B}\right)} \tag{84}$$

$$\overline{h}\times\overline{E}=-\overline{j}_{gen}+\frac{\left(\overline{h}\cdot\overline{j}_{gen}\right)}{\left(\overline{h}\cdot\overline{B}\right)}\overline{B}=\left|\overline{j}_{gen}\right|\left(-n_{\overline{j}_{gen}}+\frac{\cos\alpha}{\cos\beta}n_B\right) \tag{85}$$

being $\alpha$ the angle between the vector torsion $\overline{h}$ and the generalized current $\overline{j}_{gen}$and $\beta$ the angle between $\overline{h}$ and the magnetic field $\overline{B}$.Above, $n_B$ and $n_{\overline{j}_{gen}}$ are unitary vectors in the direction of $\overline{B}$ and $\overline{j}_{gen}$ respectively. Notice the important fact that the RHS of (68) is independent of the torsion and the magnetic field. Consequently we obtain

$$\overline{\nabla}^2\overline{B}+\underbrace{\overline{\nabla}\times\left[-\overline{j}_{gen}+\left(\frac{\left(\overline{h}\cdot\overline{j}_{gen}\right)}{\left(\overline{h}\cdot\overline{B}\right)}-h^0\right)\overline{B}\right]}_{\mathcal{E}_{Geom}}=0 \tag{86}$$

We introduce the explicitly the physical scenario via the generalized current $\overline{j}_{gen}$

$$-\overline{j}_{gen}\sim\sigma_{ext}\left[\overline{E}+\overline{v}\times\overline{B}\right]+\left(\frac{c}{e}\frac{\overline{\nabla}p}{n_e}\right) \tag{87}$$

then

$$\overline{\nabla}^2\overline{B}+\underbrace{\overline{\nabla}\times\left[\sigma_{ext}\left[\overline{E}+\overline{v}\times\overline{B}\right]+\left(\frac{c}{e}\frac{\overline{\nabla}p}{n_e}\right)+\left(\frac{\left(\overline{h}\cdot\overline{j}_{gen}\right)}{\left(\overline{h}\cdot\overline{B}\right)}-h^0\right)\overline{B}\right]}_{\mathcal{E}_{Geom}}=0 \tag{88}$$

$$\overline{\nabla}^2\overline{B}+\underbrace{\sigma_{ext}\left[\left(-\partial_t\overline{B}\right)+\overline{\nabla}\times\left(\overline{v}\times\overline{B}\right)\right]+\overline{\nabla}\times\left(\frac{c}{e}\frac{\overline{\nabla}p}{n_e}\right)+\overline{\nabla}\times\left(\frac{\left(\overline{h}\cdot\overline{j}_{gen}\right)}{\left(\overline{h}\cdot\overline{B}\right)}-h^0\right)\overline{B}}_{\mathcal{E}_{Geom}}=0 \tag{89}$$

finally the expected geometrically induced expression is obtained:

$$\partial_t\overline{B}=\eta\overline{\nabla}^2\overline{B}+\overline{\nabla}\times\left(\overline{v}\times\overline{B}\right)+\eta\overline{\nabla}\times\left[\left(\frac{c}{e}\frac{\overline{\nabla}p}{n_e}\right)+\alpha\overline{B}\right]=\partial_t\overline{B} \tag{90}$$

$$\rightarrow \eta \underbrace{\overline{\nabla}^2\overline{B}}_{diffusive} + \underbrace{\overline{\nabla} \times \left(\overline{v} \times \overline{B}\right)}_{advective} + \eta\underbrace{\overline{\nabla} \times \left(\alpha\overline{B}\right)}_{\alpha-term} + \underbrace{\frac{c}{e}\frac{\overline{\nabla}p \times \overline{\nabla}n_e}{n_e^2}}_{Biermann\ battery} = -\partial_t\overline{B} \quad (91)$$

where $\eta \equiv \frac{1}{\sigma_{ext}}$ as usual and the geometric $\alpha$:

$$\begin{aligned}\alpha &\equiv \left(\frac{\left(\overline{h}\cdot\overline{j}_{gen}\right)}{\left(\overline{h}\cdot\overline{B}\right)} - h^0\right) \\ &= \left(\frac{\cos\alpha\left|\overline{j}_{gen}\right|}{\cos\beta\left|\overline{B}\right|} - h^0\right)\end{aligned} \quad (92)$$

### 8.5. Seed magnetic field

Notice from the last expression that $\alpha\overline{B}$ is explicitly

$$\alpha\overline{B} = \frac{\cos\alpha\left|\overline{j}_{gen}\right|}{\cos\beta}n_B - h^0\overline{B} \quad (93)$$

or (via elimination of the unitary vector)

$$\alpha\left|\overline{B}\right| = \frac{\cos\alpha\left|\overline{j}_{gen}\right|}{\cos\beta} - h^0\left|\overline{B}\right| \quad (94)$$

we see *clearly* the first term in RHS independent of the intensity of the magnetic field. Considering only the terms of interest without the diffusive and advective term in the induction equation(only time-dependence for the magnetic field is preserved) namely

$$\underbrace{\eta\overline{\nabla} \times \left(\alpha\overline{B}\right)}_{\alpha-term} = -\partial_t\overline{B} \quad (96)$$

$$\eta\overline{\nabla}\left(\frac{\cos\alpha\left|\overline{j}_{gen}\right|}{\cos\beta}\right) = -\partial_t\left|\overline{B}\right| \quad (97)$$

we see that the currents given by the fields (related to the geometry via $h_\alpha$) originate the magnetic field.

If we consider all the currents of the fields of theory (fermions, bosons, etc.) the seed would be precisely these field currents. The other missing point is to derive the fluid (hydrodynamic) equations (which as is known does not have a definite Lagrangian formulation) from the same unified formulation. Notice that there are, under special conditions, analogous formulas for vorticity $\omega$ than for the magnetic field $B$. This would mean that the 2-form of vorticity must also be included in the fundamental antisymmetric tensor, together with the electromagnetic field.

### 8.6. Comparison with the mean field formalism

Now we compare the obtained equations with respect to the mean field formalism. Starting from expressions (69-72) as before, we have:

$$\eta\overline{\nabla}^2\overline{B} + \overline{\nabla} \times \left(\overline{v} \times \overline{B}\right) - \partial_t\overline{B} + \underbrace{\eta\overline{\nabla} \times \left(-h^0\overline{B} + \overline{h} \times \overline{E}\right)}_{\mathcal{E}_{Geom}} = 0 \qquad (98)$$

$\mathcal{E}_{Geom}$ takes the place of electromotive force due the torsion field with full analogy as $\mathcal{E} = \langle u \times b\rangle$ is the mean electromotive force due to fluctuations. Also as in the mean field case that there are the splitting

$$\mathcal{E} = \mathcal{E}^{\langle 0\rangle} + \mathcal{E}^{\langle \overline{B}\rangle} \qquad (99)$$

with $\mathcal{E}^{\langle 0\rangle}$ independent of $\left\langle\overline{B}\right\rangle$ and $\mathcal{E}^{\langle \overline{B}\rangle}$linear and homogeneous in $\overline{B}$, we have in the torsion case the following correspondence

$$-h^0\overline{B} \longleftrightarrow \mathcal{E}^{\langle \overline{B}\rangle}$$
$$\overline{h} \times \overline{E} \longleftrightarrow \mathcal{E}^{\langle 0\rangle}$$
$$geometrical \longleftrightarrow turbulent$$

Consequently, the problems of mean-field dynamo theory that are concerned with the generation of a mean EMF by turbulence, have in this model a pure geometric counterpart. In the past years, attention has shifted from kinematic calculations, akin to those familiar from quasilinear theory for plasmas, to self-consistent theories which account for the effects of small scale magnetic fields (including their back-reaction on the dynamics) and for the constraints imposed by the topological conservation laws, such as that for magnetic helicity. Here the torsion vector generalize (as we can see from above set of equations) the concept of helicity. The consequence of this role of the dual torsion field is that the traditionally invoked mean-field dynamo mechanism (i.e. the so-called alpha effect) may be severely quenched or increased at modest fields and magnetic Reynolds numbers, and that spatial transport of this generalized magnetic helicity is crucial to mitigating this quench. Thus, the dynamo problem is seen in our model as one of generalized helicity transport, and so may be tackled like other problems in turbulent transport. A key element in this approach is to understand the evolution of the torsion vector field besides of the turbulence energy and the generalized helicity profiles in space-time. This forces us to confront the problem of spreading of strong MHD turbulence, and a spatial variant or analogue of the selective decay problem with the dynamics of the torsion field.

## 9. Torsion, axion electrodynamics vs. Chern Simons theory

Let us review briefly the electromagnetic sector of the theory QCD based in a gauge symmetry $SU(3) \times U(1)$

$$L_{QCD/QED} = +\sum \overline{\psi}_f \left[\gamma^\mu \left(\partial_\mu - ig_f t^\alpha A^\alpha_\mu - iq_f A_\mu\right) - m_f\right] \psi_f - \quad (100)$$
$$-\frac{G^\alpha_{\mu\nu} G^{\alpha\mu\nu}}{4} - \frac{F_{\mu\nu}F^{\mu\nu}}{4} - \frac{g^2 \theta G^\alpha_{\mu\nu} \widetilde{G}^{\alpha\mu\nu}}{32\pi^2} - \frac{g^2 \theta F_{\mu\nu} \widetilde{F}^{\mu\nu}}{32\pi^2},$$

As is well know, electromagnetic fields will couple to the electromagnetic currents, namely:$J_\mu = \sum_f q_f \overline{\psi}_f \gamma_\mu \psi_f$ consequently , there appear term will induce through the quark loop the coupling of $F_{\mu\nu}\widetilde{F}^{\mu\nu}$ (the anomaly) to the QCD topological charge . The effective Lagrangian can be written as

$$L_{MCS} = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} - A_\mu J^\mu - \frac{c}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu} \quad (101)$$

where a pseudo-scalar field $\theta = \theta(\overline{x}, t)$ (playing the role of the axion field) is introduced and $c = \sum_f \frac{(q_f e)^2}{2\pi^2}$ . This is the Chern-Simons Lagrangian where, if $\theta$is constant, the last term is a total divergence: $F_{\mu\nu}\widetilde{F}^{\mu\nu} = \partial_\mu J^\mu_{CS}$.The question appear if $\theta$ is not a constant $\theta F_{\mu\nu}\widetilde{F}^{\mu\nu} = \theta\partial_\mu J^\mu_{CS} = \partial_\mu \left(\theta J^\mu_{CS}\right) - J^\mu_{CS}\partial_\mu\theta$

Now we can see from the previous section that if, from the general decomposition of the four dimensional dual of the torsion field via the Hodge de Rham theorem we retain $b_\alpha$ as gradient of a pseudoscalar (e.g: axion) these equations coincide in form with the respective equation for MCS theory. Precisely because under this condition $h_\alpha = \nabla_\alpha \theta$ , in flat space (curvature=0 but torsion$\neq 0$) the equations become the same as in namely

$$\overline{\nabla} \cdot \overline{E} - c\overline{P} \cdot \overline{B} = \rho_{ext} \quad (102)$$
$$\partial_t \overline{E} - \overline{\nabla} \times \overline{B} = -c\dot{\theta}\overline{B} + c\overline{P} \times \overline{E} - \sigma_{ext}\left[\overline{E} + \overline{v} \times \overline{B}\right] \quad (103)$$
$$\overline{\nabla} \cdot \overline{B} = 0 \quad (104)$$
$$\partial_t \overline{B} = -\overline{\nabla} \times \overline{E} \quad (105)$$

provided:

$$h^0 \to -c\dot{\theta} \quad (106)$$
$$\overline{h} \to -c\overline{P} \quad (107)$$

where from QCD the constant $c$ is determined as $c = \frac{e^2}{2\pi}$ and the $\partial_\mu \theta = \left(\dot{\theta}, \overline{P}\right)$ . The main difference is that while in the case of photons in axion ED was given by the Lagrangian where that above equations are derived is

$$L_{MCS} = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} - A_\mu J^\mu + \frac{c}{4} P_\mu J^\mu_{CS}, \qquad J^\mu_{CS} \equiv \varepsilon^{\mu\sigma\rho\nu}\, A_\sigma F_{\rho\nu} \tag{108}$$

in our case is the dual of the torsion field (that we take as the gradient of a pseudoscalar) responsible of the particular structure of the set of equations.

## 10. Magnetic helicity generation and cosmic torsion field

Here we consider the projective invariant case: $\beta = 0$ ($R_A = -4\lambda$)where the gravitational and field equations are considerably simplified because $\mathbb{R} = 1$ and $b^{-1} = 0$ . Scalar curvature $R$ and the torsion 2-form field $T^a_{\mu\nu}$ with a $SU(2)$ $-$Yang-Mills structure are defined in terms of the affine connection $\Gamma^\lambda_{\mu\nu}$ and the $SU(2)$ valuated (structural torsion potential) $f^a{}_\mu$ by

$$R = g^{\mu\nu} R_{\mu\nu} \qquad R_{\mu\nu} = R^\lambda_{\mu\lambda\nu} \tag{109}$$

$$R^\lambda_{\mu\lambda\nu} = \partial_\nu \Gamma^\lambda_{\mu\rho} - \partial_\rho \Gamma^\lambda_{\mu\nu} + ...$$

$$T^a{}_{\mu\nu} = \partial_\mu f^a{}_\nu - \partial_\nu f^a{}_\mu + \varepsilon^a_{bc} f^b{}_\mu f^c{}_\nu$$

$G$ and $\Lambda$ are the geometrically induced Newton gravitational constant (as we have been discussed before) and the integration cosmological constant, respectively. From the last equation for the totally antisymmetric Torsion 2-form, the potential $f^a{}_\mu$ define the affine connection $\Gamma^\lambda_{\mu\nu}$ .Similarly to the case of Einstein-Yang-Mills systems, for our new $UFT$ model it can be interpreted as a prototype of gauge theories interacting with gravity (*e.g.* QCD, GUTs, etc.). We stress here the important fact that all the fundamental constants are really geometrically induced as required by the Mach principle. After varying the action, we obtain the gravitational equation (41), namely

$$\overset{\circ}{R}_{\alpha\beta} - \frac{g_{\alpha\beta}}{2} \overset{\circ}{R} = 6\left(-h_\alpha h_\beta + \frac{g_{\alpha\beta}}{2} h_\gamma h^\gamma\right) + \kappa_{geom}\left[F_{\alpha\lambda} F_\beta{}^\lambda - F_{\mu\nu}F^{\mu\nu}\frac{g_{\alpha\beta}}{4}\right] + \Lambda g_{\alpha\beta} \tag{110}$$

with the "gravitational constant" geometrically induced as

$$\kappa_g \equiv \frac{R_s}{2\mathcal{W}} = \left.\frac{8\pi G}{c^4}\right|_{today} \tag{111}$$

and the field equation for the torsion 2-form in differential form

$$d^*T^a + \tfrac{1}{2}\varepsilon^{abc}\left(f_b \wedge^* T_c -^* T_b \wedge f_c\right) = -\lambda\, {}^*f^a \tag{112}$$

Notice that $\kappa_g$ and $\Lambda$ are not independent, but related by $R_s = 2\Lambda$. In this case $\beta = 0$ we have the simplest expression:

$$\kappa_g \equiv \frac{R_s}{2\left(1+\frac{R_s}{4\lambda}\right)^2} = \frac{\Lambda}{\left(1+\frac{2\Lambda}{4\lambda}\right)^2}$$

in consequence, generalizing the conjecture of Markov **if $\Lambda$ is proportional to the energy,** $\kappa$ goes as $\Lambda$ if $|\Lambda| \leq 1$, and as $\Lambda^{-1}$ in other case.

We are going to seek for a classical solution with the following ansatz for the metric and gauge connection

$$ds^2 = d\tau^2 + a^2(\tau)\,\sigma^i \otimes \sigma^i \equiv d\tau^2 + e^i \otimes e^i. \tag{113}$$

Here $\tau$ is the euclidean time and the dreibein is defined by $e^i \equiv a(\tau)\,\sigma^i$. The gauge connection is

$$f^a \equiv f^a_\mu dx^\mu = f\sigma^a, \tag{114}$$

for $a, b, c = 1, 2, 3$, and for $a, b, c = 0$ we have

$$f^0 \equiv f^0_\mu dx^\mu = s\sigma^0. \tag{115}$$

This choice for the potential torsion is accordingly to the symmetries involved in the problem.

The $\sigma^i$ 1-form satisfies the $SU(2)$ Maurer-Cartan structure equation

$$d\sigma^a + \varepsilon^a{}_{bc}\sigma^b \wedge \sigma^c = 0 \tag{116}$$

Notice that in the ansatz the frame and $SU(2)$ (isospin-like) indices are identified (as for the case with the non-abelian-Born-Infeld (NBI) Lagrangian ) The torsion 2-form

$$T^\gamma = \frac{1}{2}T^\gamma{}_{\mu\nu}dx^\mu \wedge dx^\nu \tag{117}$$

becomes

$$\begin{aligned} T^a &= df^a + \frac{1}{2}\varepsilon^a{}_{bc}f^b \wedge f^c \\ &= \left(-f + \frac{1}{2}f^2\right)\varepsilon^a{}_{bc}\sigma^b \wedge \sigma^c \end{aligned} \tag{118}$$

$$\begin{array}{rcl} d^*T^a + \frac{1}{2}\varepsilon^{abc}\left(f_b \wedge^* T_c -^* T_b \wedge f_c\right) &=& -2\lambda\, {}^*f^a \\ (-2f + f^2)(1-f)d\tau \wedge e^b \wedge e^c &=& -2\lambda d\tau \wedge e^b \wedge e^c \end{array} \tag{119}$$

$${}^*T^a \equiv h(-2f + f^2)d\tau \wedge \frac{e^a}{a^2} \tag{120}$$

$$^*f^a = -f\frac{d\tau \wedge e^b \wedge e^c}{a^3} \tag{121}$$

Note that to be complete in our description of the possible physical scenarios, we include $f^0$ as an $U(1)$ component of the torsion potential (although does not belong to the space $SU(2)/U(1)$). Having all the above issues into account, the expression for the torsion is analogous to the non-abelian 2-form strength field.

Inserting $T^a$ from (118) into the dynamic equation (112) we obtain

$$(-2f + f^2)(1-f)d\tau \wedge e^b \wedge e^c = -\lambda d\tau \wedge e^b \wedge e^c, \tag{122}$$

and from expression (122) we have an algebraic cubic equation for $f$

$$(-2f + f^2)(1-f) + \lambda = 0 \tag{123}$$

We can see that, in contrast with our previous work with a dualistic theory where the NBI energy-momentum tensor of Born-Infeld was considered, there exist three non trivial solutions for $f$, depending on the cosmological constant $\lambda$. In this preliminary analysis of the problem, only the values of $f$ that make the quantity $\left(-f + \frac{1}{2}f^2\right) \in \mathbb{R}$. Consequently for $\lambda = 2$ we find $f = 2.35$ then

$$T^a_{bc} = \frac{2}{5}\frac{\varepsilon^a_{bc}}{a^2}; \qquad\qquad T^a_{0c} = 0 \tag{124}$$

That is, only spatial torsion field is non vanishing while cosmic time torsion field vanishes. Substituting the expression for the torsion 2-form [1] into the symmetric part of the variational equation we reduce the gravitational equations to an ordinary differential equation for the scale factor $a$,

$$3\left[\left(\frac{\dot a}{a}\right)^2 - \frac{1}{a^2}\right] - \Lambda = \frac{3\kappa_g}{4a^2}\left(f^2 + s^2\right) + \frac{3}{2a^4}f^2\left(f-2\right)^2 \tag{125}$$

that in the case for the computed value for $f \sim 2.35$ with $s = 10$ and $\Lambda \lesssim 1$ the scale factor is described in Figure 1 and the scale factor goes as:

$$a\left(\tau\right) = \Lambda^{-1/2}\sqrt{\left(1 - \frac{12\kappa_g^2\Lambda}{\alpha}\right)^{1/2}\sinh\left(\sqrt{\Lambda/3}\left(\tau - \tau_0\right)\right) - 1 + \kappa_g\left(f^2 + s^2\right)/4} \tag{126}$$

where we define the geometrically induced fine structure function $\alpha \equiv \kappa_g\left(f^2 + s^2\right)/4$

[1] in the tetrad: $\overset{\circ}{R}_{00} = -3\frac{\ddot a}{a}, \overset{\circ}{R}_{ab} = -\left[\frac{\ddot a}{a} + 2\left(\frac{\dot a}{a}\right)^2 - \frac{2}{a^2}\right]$

### 10.1. Primordial symmetries of standard model and torsion field

Recently (see ref. in [1][2][4]the cross section for neutrino helicity spin flip obtained from this type of $f(R;T)$ model of gravitation with dynamic torsion field introduced by us was phenomenologically analyzed using the relation with the axion decay constant $f_a$ (Peccei-Quinn parameter) due the energy dependence of the cross section, Consequently, the link with the phenomenological energy/mass window was found from the astrophysical and high energy viewpoints. The important point is that, in relation with the torsion vector interaction Lagrangian, the $f_a$ parameter gives an estimate of the torsion field strength that can variate with time within cosmological scenarios as the described above, potentially capable of modifying the overall leptogenesis picture, the magnetogenesis, the bariogenesis and also to obtain some indication about the primordial (super) symmetry of the early universe.

In FRW scenario given here we saw that the torsion through its dual vector, namely:

$$h^0 = \frac{2}{5}\frac{\delta_a^0 C_\tau}{a^2} d\tau \wedge e^a \tag{127}$$

goes as $\sim a^{-2}$ with $C_\tau$ a covariantly constant vector field $\left(\text{e.g.:}\overset{\circ}{\nabla} C_\tau = 0\right)$ that we take of the form $C_\tau \sim \left(\dot{\theta} + q_\tau\right)$ (due the Hodge-de Rham decomposition of $h_\mu$, expression(42))where $\theta$is a pseudoscalar field playing the obvious role of axion and $q_\tau$ :vector field linking $h^0$ with the magnetic field via the equation of motion for the torsion. Consequently, the torsion dual vector $h^i$has the maximum value when the radius of the universe is $a_{\min}$ , e.g. $a_{\min} = a(\tau_0)$ increases to the maximum value the spin-flip neutrino cross section and, for instance, the quantity of right neutrinos compensating consequently the actual (e.g. $a_{today} = a(\tau)$)assymetry of the electroweak sector of the SM (see the behaviour of $a$ in Fig.1). This fact indicates that the original symmetry group contains naturally $SU_R(2) \times SU_L(2) \times U(1)$ tipically inside GUT's structurally based generally in $SO(10)$, $SU(5)$ or some exceptional groups as $E(6)$,$E(7)$, *etc.*

Also it is interesting to note that from the FRW line element written in terms of the cosmic time the Hubble flow electromagnetic fields $E_\mu \equiv$

$(0, E_i) = a^{-2}(0, \partial_\tau A_i)$ and $B_\mu \equiv (0, B_i) = a^{-2}(0, \varepsilon_{ijk}\partial_j A_k)$

$$\overline{\nabla}\cdot\overline{E} + \left(\frac{\alpha}{f}\overline{\nabla}\theta + \overline{\Pi}\right)\cdot\left(a^2\overline{B}\right) = 0 \tag{111}$$

$$\partial_\tau\left(a^2\overline{E}\right) - \overline{\nabla}\times\left(a^2\overline{B}\right) = \left(\frac{\alpha}{f}\partial_\tau\theta + \Pi_0\right)\left(a^2\overline{B}\right) - \left(\frac{\alpha}{f}\overline{\nabla}\theta + \overline{\Pi}\right)\times\overline{E} \tag{112}$$

$$\overline{\nabla}\cdot\overline{B} = 0 \tag{113}$$

$$\partial_t\overline{B} = -\overline{\nabla}\times\overline{E} \tag{114}$$

where $\Pi_\mu \equiv f_\mu\left(u_\mu, \gamma^5 b_\mu, eA_\mu .....\right)$ is a vector function of physical entities as potential vector, vorticity, angular velocity, axial vector etc etc. as described by expression (42). In principle we can suppose that it is zero (low back reaction ) then

$$\overline{h} = \frac{\alpha}{f}\overline{\nabla}\theta, \qquad h^0 = \frac{\alpha}{f}\partial_\tau\theta \tag{115}$$

being $\left[\partial_\tau^2 - \overline{\nabla}^2 - \frac{\alpha}{f}\partial_\tau\theta\overline{\nabla}\times\right]\left(a^2\overline{B}\right) = 0$ the second order equation for the magnetic field that shows the chiral character of the plasma particles.

## 10.2. Magnetogenesis and cosmic helicity

Now we pass to see which role plays the torsion field in the magnetic field generation in a FRW cosmology. Taking as the starting point the (hyper) electrodynamic equations and introducing a Fourier mode decomposition $\overline{B}(\overline{x}) = \int d^3\overline{k}B\left(\overline{k}\right)e^{-i\overline{k}\cdot\overline{x}}$ with $\overline{B}\left(\overline{k}\right) = h_i\overrightarrow{e}_i$ where $i = 1, 2$, $\overrightarrow{e}_i^2 = 1$, $\overrightarrow{e}_i\cdot\overrightarrow{k} = \overrightarrow{e}_1\cdot\overrightarrow{e}_2 = 0$ the torsion-modified dynamical equations for the expanding FRW become

$$\dot{\overline{z}} + \left[\left(2\dot{a} + \frac{k^2}{\sigma}\right) + \frac{ah^0\left|k\right|}{\sigma}\right]\overline{z} = 0 \tag{116}$$

$$\dot{z} + \left[\left(2\dot{a} + \frac{k^2}{\sigma}\right) - \frac{ah^0\left|k\right|}{\sigma}\right]z = 0 \tag{117}$$

where the magnetic field is written in terms of complex variable $z\left(\overline{z}\right)$ as

$$z = h_1 + ih_2 \tag{118}$$

$$\overline{z} = h_1 - ih_2 \tag{119}$$

from equation (117) we see that the solution for $z$ namely:

$$z = z_0 e^{-\left(2a + \frac{k^2}{\sigma}\tau\right) + \int\frac{ah^0|k|}{\sigma}d\tau} \tag{120}$$

contains the instable mode in the sense of $\frac{k}{\sigma}\tau < \int \frac{ah^0}{\sigma} d\tau$. Consequently a defined polarization of the magnetic field appear and from the dynamical equation for the torsion field: $\nabla_{[\mu} h_{\nu]} = -\lambda \widetilde{F}_{\mu\nu}$ that in this case we have

$$\nabla_{[i} h_{\tau]} = \nabla_i \left(a^{-2} q_\tau\right) = -\lambda B_i \tag{121}$$

that implies a relation between the vector part of the $h^0$ (namely $q_\tau$) with the vector potential $A^k$ of the magnetic field as follows:

$$\nabla_i q_\tau \approx -\lambda \varepsilon_{ijk} \nabla^j A^k \tag{122}$$

Consequently, the primordial magnetic field (or seed) would be connected in a self-consistent way with the torsion field by means of the dual vector $h_0$.It $(h_\mu)$ in turn, would be connected phenomenologically with the physical fields (matter) of theory through Hodge-de Rham decomposition expression (42). We note from expression (120) that the pseudo-scalar (axion) controls the stability, growth and dynamo effect but not the generation of the magnetic field (primordial or seed) as is clear from expression (122) where the (pseudo) -vector part of $h_0$ contributes directly to the generation of the magnetic field as clearly given by eq. (121)

### 10.3. Magnetogenesis and cosmic helicity II

In the case to include the *complete alpha term* given by equations (92) and in the same analytical conditions (e.g.: Fourier decomposition) from the previous paragraph, the torsion-modified dynamic equations for the expanding FRW become

$$\dot{\overline{z}} + \left[\left(2\dot{a} + \frac{k^2}{\sigma}\right) + \frac{a\,|k|}{\sigma}\left(h^0 - \frac{\cos\alpha\,\left|\overline{j}_{gen}\right|}{\cos\beta\,|\overline{z}|}\right)\right]\overline{z} = 0 \tag{123}$$

$$\dot{z} + \left[\left(2\dot{a} + \frac{k^2}{\sigma}\right) - \frac{a\,|k|}{\sigma}\left(h^0 - \frac{\cos\alpha\,\left|\overline{j}_{gen}\right|}{\cos\beta\,|z|}\right)\right] z = 0 \tag{124}$$

where in this case the magnetic field is written (by convenience) in terms of complex variable $z\,(\overline{z})$ as

$$z = |z|\, e^{i\rho} \to \dot{z} = \left(\dot{|z|} + i\dot{\rho}\,|z|\right) e^{i\rho} \tag{125}$$

$$\overline{z} = |\overline{z}|\, e^{-i\rho} \to \dot{\overline{z}} = \left(\dot{|\overline{z}|} - i\dot{\rho}\,|\overline{z}|\right) e^{-i\rho} \tag{126}$$

From equation (124) we see that the solution for $z$ namely:

$$z = z_0 \exp\left[-\left(2a + \frac{k^2}{\sigma}\tau\right) + \int \frac{a\,|k|}{\sigma}\left(h^0 - \frac{\cos\alpha\,\left|\overline{j}_{gen}\right|}{\cos\beta\,|z_0|}\right)d\tau\right] \tag{127}$$

with $z_0 = |z_0|\,e^{i\rho_0}$ $\qquad(|z_0| = const)$

contains the instable mode in the sense of Joice and Shaposchnikov, for example (117)$\frac{k}{\sigma}\tau < \int \frac{a}{\sigma}\left(h^0 - \frac{\cos\alpha\left|\overline{j}_{gen}\right|}{\cos\beta|z|}\right)d\tau$. But now *there are not a definite polarization for the magnetic field*, but now all depends on the difference:

$$\int \frac{a}{\sigma}\left(h^0 - \frac{\cos\alpha\,\left|\overline{j}_{gen}\right|}{\cos\beta\,|z_0|}\right)d\tau$$

Replacing explicitly $h_\alpha$ from the decomposition (42) we can see in a clear form, the interplay between the physical entities, as the vortical and magnetic helicities for example:

$$\left(\nabla_0\Omega + \frac{4\pi}{3}\left[h_M + q_s n_s \overline{u}_s \cdot \overline{B}\right] + \gamma_1 h_V + \gamma_2 P_0\right) - \frac{\cos\alpha\,\left|\overline{j}_{gen}\right|}{\cos\beta\,|z_0|}$$

Now considering in $\left|\overline{j}_{gen}\right|$the fermionic current $\sum_f q_f\overline{\psi}_f\gamma_\mu\psi_f$ , $\Omega$ as the axion $a$ , $|z_0| = \frac{\cos\beta}{\cos\alpha}$and putting $\gamma_2 = 0$ we have an interesting expression:

$$\nabla_0 a + \frac{4\pi}{3}\left[h_M + q_s n_s \overline{u}_s \cdot \overline{B}\right] + \gamma_1 h_V = \left|\sum_f q_f\overline{\psi}_f\gamma_\mu\psi_f\right|$$

The above expression it is very important because establishes the desired connection between helicities, magnetic field and fermionic fields and axion. We can order it as

$$\nabla_0 a - \left|\sum_f q_f\overline{\psi}_f\gamma_\mu\psi_f\right| = -\left[\frac{4\pi}{3}\left(h_M + q_s n_s \overline{u}_s \cdot \overline{B}\right) + \gamma_1 h_V\right]$$

We now clearly see the link between the axion and the fermionic fields (the dynamics of the interacting fields and the involved currents) in the LHS and the macroscopic physical observables in the RHS giving an indication of the origin of leptogenesis and bariogenesis in the context of this non Riemannian gravitational model.

## 11. Discussion and perspectives

In this paper we have introduced a simple geometric Lagrangian in the context of a unified theory based on affine geometry. From the functional action proposed, that is as square root or measure, the dynamic equations were derived: an equation analogous to trace free Einstein equations $TFE$ and a dynamic equation for the torsion (which was taken totally antisymmetric). Although the aim of this paper was to introduce and to analize the model from the viewpoint of previous research, we bring some new results and possible explanations about the generation of primordial magnetic fields and the link with the leptogenesis and baryogenesis. The physically admisible analysis of the torsion vector $h_{\mu}$, from the point of view of the symmetries, has allowed us to see how matter fields can be introduced in the model. These fields include some dark matter candidates such as axion, right neutrinos and Majorana. Also the vorticity can be included in the same way and, as the torsion vector is connected to the magnetic field, both vorticity and magnetic field can be treated with equal footing. The other point is that from the wormhole soluton in a cosmological spacetime with torsion we show that primordial cosmic magnetic fields can be originated by the dual torsion field $h_{\mu}$ being the axion field contained in $h_{\mu}$, the responsible to control the dynamics and stability of the cosmic magnetic field, but is not responsible of the magnetogenesis itself. Also the energy conditions in the wormhole solution are fulfilled. The last important point to highlight is that the dynamic torsion field $h_{\mu}$ acts as mechanism of the reduction of an original (early, primordial) GUT (Grand Unified Theory) symmetry of the universe containing $\sim$SU(3) $\times$SU$(2)_R$ $\times$SU$(2)_L$ $\times$U(1) to SU(3) $\times$SU$(2)_L$ $\times$U(1) today. Consequently, the GUT candidates are $SO(10)$, $SU(5)$ or some exceptional groups as $E(6)$,$E(7)$ for example. Also we can give some hints on the astrophysical consequences of the affine structure underlying the gravitational theory presented here, as the new dynamo effects manifestation by mean a modified Grad Shafranov equation, acrretion mechanisms and magnetosphere structucture driven by axion fields, etc

## 12. Acknowlegments

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