Cycles Cohomology by Integral Transforms in Derived Geometry to Ramified Field Theory

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Theorem 4.1. *One meromorphic extension of one flat connection given through a Hitchin construction we can give the following commutative co-cycles diagram to the category* $M_{\mathcal{K}_P}(\hat{\mathfrak{g}}, Y)$*,*

$$\begin{array}{ccccc}
\mathbf{h}\epsilon H^0(T^{\vee}Bun_G, \mathcal{D}^s) & \xrightarrow{d} & H^1(T^{\vee}Bun_G, \mathcal{O}) & \xrightarrow{\cong} & \Omega^1[\mathbf{H}] \\
\cong\downarrow & & \cong_{\Phi\mu}\downarrow & & \downarrow\pi \\
a\epsilon\mathbb{C}[Op_{L_G}] & \xrightarrow{d} & \Omega^1[\mathcal{O}_{Op_{L_G}}] & \xrightarrow{d} & C \times B
\end{array} \qquad (4)$$

[F. Bulnes, TMA, UK, 2017]

We establish the following hypothesis:
1. Adequated Moduli Problem:

Theorem 3.1. (F. Bulnes) [5]. *If we consider the category* $M_{\mathcal{K}_F}(\hat{\mathfrak{g}}, Y)$, *then a scheme of their spectrum* $V^{Def}_{critical}$, *where* Y, *is a Calabi-Yau manifold comes given as:*

$$Hom_{\hat{\mathfrak{g}}}(X, V^{Def}_{critical}) \cong Hom_{Loc_{LG}}(V_{critical}, M_{\mathcal{K}_F}(\hat{\mathfrak{g}}, Y)), \qquad (1)$$

Proof. [5].

[F. Bulnes, PAMJ, 2014, F. BULNES, TMA, 2015]

2. Extension of Commutative Rings:

Theorem 3.2. *The Yoneda algebra* $Ext_{\mathcal{D}^s(Bun_G)}(\mathcal{D}^s, \mathcal{D}^s)$, *is abstractly* A_∞*-isomorphic to* $Ext^{\bullet}_{Loc_{LG}}(\mathcal{O}_{Op_{LG}}, \mathcal{O}_{Op_{LG}})$.

Proof. [6],[7].

Formal Deformations of Sheaves can extended to deformation of categories to QFT

4. Twisted Nature of the Derived Categories $D^{\times}$ on an appropiate stack:

Lemma (F. Bulnes) 3. **1.** *Twisted derived categories corresponding to the algebra of functions $\mathbb{C}[Op_{^LG}(D^{\times})]$, are the images obtained by the composition $\mathcal{P}(\tau)$, on $\tilde{\mathcal{L}}_{\lambda}, \forall \lambda \epsilon \mathfrak{h}^*$, and such that their Penrose transform is:*

$$\mathcal{P} : H^0({}^LG, \Gamma(Bun_G, \mathcal{D}^{\times})) \cong Ker(U, \tilde{\mathcal{D}}_{\lambda,y}), \quad (2)$$

[F. Bulnes, ILIRIAS ZbMATH, 2014]

NOTE: Here is exhibited the Cohomology space $H^{\bullet}(?, \Omega^{\bullet})$, as the space $H^{\bullet}(\mathrm{H}^{\vee}, \Omega^{\bullet})$. [F. Bulnes, TMA, 2016]

Inside of quasi-coherent category given by $M_{\mathcal{K}_F}(\hat{\mathfrak{g}}, Y)$ [F. Bulnes, APM, 2013]

which carry us to the ramification problem.

5. We consider QFT AND TFT in the Derived Categories frame to define de co-cycles of $M_{\mathcal{K}_F}(\hat{\mathfrak{g}}, Y)$

Then field ramifiations are connections of $\Omega^{\bullet}(Op_{L_G}(D))$

Proof **(Mein Theorem or Theorem 4. 1).**

-First we demonstrate the equality between $R^1\chi_*(\mathcal{D}^s)$ **and** $R^1\chi_*(\mathrm{h})\epsilon\Omega^1[\mathrm{H}]$, **(diagram) in the derived categories class** $\mathcal{D}^{\times}$**.**

$$\begin{array}{ccccc} & & R^1\chi_* & & \\ & & \longrightarrow & & \\ | & & & & \downarrow \\ H^0(\Sigma,\Omega^1) & \xrightarrow{\Gamma} & H^1(\Sigma,\Omega^2) & \xrightarrow{\cong} & \Omega^1[\mathrm{H}] \\ \cong\downarrow & & \cong\downarrow & & \downarrow \pi_{\mathrm{H}} \\ \Omega^1(\Sigma^0,\mathfrak{g}) \xrightarrow{d} & & \Omega^2(\Sigma,\mathfrak{g}) & & \xrightarrow{a} C\times B \end{array}$$

We consider a Langlnds correspondence such that: $\Phi^i(\mathfrak{c}(\mathcal{O}_v)) = \mathcal{O}(\mathcal{O}_v) \boxtimes \wedge^i \mathbb{V}$, arriving to $\Omega^i[Op_{L_G}]$ which gives the equivalence of complexes:

$$\{d\mathbf{h} = 0\} \cong^{L\Phi^\mu} \{da = 0\},$$

which required the correspondence

$$\tilde{\mathfrak{c}} : D_{Coh}(T^\vee Bun_G, \mathcal{O}) \cong D_{Coh}(^L T^\vee Bun_G, \mathcal{O}),$$

[F. Bulnes, scirp, USA, 2016]

Then is demonstrated the first decendant Isomorphism:

$$\mathbf{h} \epsilon H^0(T^\vee Bun_G, \mathcal{D}^s)$$

$$\cong\downarrow$$

$$a \epsilon \mathbb{C}[Op_{L_G}]$$

NOTE:

$$\tilde{\mathfrak{c}} = \mathfrak{c}(T^\vee \mathcal{O}_{Op_{L_G}}) = \mathfrak{c}(\mathcal{O}^{DG}_{SP_u}),\ ^6\text{such that } \Phi_{\mathcal{O}^{DG}_{Spu}}(\mathbb{C}_u) \cong \Gamma q_{2!}(\mathcal{O}^L_{\mathcal{N}} \otimes_{\mathcal{N}} \mathbb{C}_u).$$

[F. Bulnes, JMSS, 2013]

By Frenkel equivalence:

$$D^b(\mathfrak{g}_{KC} - mod_{nilp})^{I^0} \cong D^b(QCoh(MOp^{nilp}_{^LG})),$$

Each quasi-coherent sheaf on the kernel of the right side of the before equivalence corresponds an object of $D^b(M_{\mathcal{K}_c}(\tilde{\mathfrak{g}}, Y))^{I_0}$ then:

a). The functor $\Phi_{\mathcal{O}^{DG}_{Spu}}$ is the Hecke functor.
b). It´s integral transform such that

$$h : Bun_{Higgs} \rightarrow B \text{ (to a quantized))}$$

And equivalent to $D_{coh}(^LBun, \mathcal{D})$

Then the geometric Langlands conjecture in terms of Higgs bundles, consider a functor between the categories $D_{coh}(^LLoc, \mathcal{O})$, with the action of the Hecke functors on $D_{coh}(^LBun, \mathcal{D})$.

But $MOp_{^LG}^{nilp} \cong Op_{^LG}^{nilp} \times \tilde{\mathcal{N}} / {}^LG$ **by Steinberg manifold structure to Langlands Correspondence given by** $\mathfrak{c}$, **such that:**

$\tilde{\mathfrak{c}} = \mathfrak{c}(\mathcal{O}_{Sp_uC}(\mathbb{V}) \times \mathbb{C}^{\times})$, where $\mathcal{O}_{Sp_uC} = \mathcal{O}_{\tilde{\mathcal{N}}} \otimes_{\mathcal{O}\mathcal{N}} \mathbb{C}$, and $\mathcal{N} \subset {}^L\mathfrak{g}$,

By K-theory, the Steinberg variety who have Elements $C \times B$, **that satisfy:**

$Isom(d\mathbf{h}) = d(da), \forall a \in \mathbb{C}[Op_{L_G}(D)]$,

Then is had that: $\{d(da)\} \overset{L}{\underset{\longleftrightarrow}{}}\Phi^{\mu} \{Isom d\mathbf{h}\}$,

In other words to the kernels of $\Omega^i, i = 1, 2, \ldots,$ are the that are in sheaf $\mathcal{O}_{Op_{L_G}}$, [10] that is to say, there is an extended Penrose transform such that the kernels set are the fields $\mathbf{h}$, with $Isom(d\mathbf{h}) = 0$,in the hyper-cohomology

Then this the hypercohomology in the down line through Hitchin mapping takes the fields

$d(da) = 0$, in the hyper-cohomology $\mathbb{H}(\Omega^1 \xrightarrow{d} \Omega^2 \xrightarrow{d} \ldots)$.

In the context of the Differential operators algebra:
We can to give the commutative rings diagram:

$$\begin{array}{ccccc} H^\bullet(\mathfrak{g}[[z]], \mathbb{V}_{critical}) & \rightarrow & H^\bullet(\mathfrak{g}[[z]], \mathfrak{g}, \mathbb{V}_{critical}) & \xrightarrow{\cong} & \Omega^\bullet[\mathbf{H}] \\ \cong\downarrow & & \cong_\Phi\downarrow & & \downarrow\pi \\ \mathbb{C}[Op_{^LG}] & \xrightarrow{d} & \Omega^\bullet[Op_{^LG}] & \xrightarrow{d} & \mathbf{H}^\vee \end{array}$$

Then we have the Penrose transform in the decendant isomorphism whose field solutions are
To the equations da = 0.
An extended version of Penrose Transform to deformed modules version $H^\bullet(\mathfrak{g}[[z]], \mathfrak{g}, \mathbb{V}_{critical})$ consider
The deformed Mukai-Fourier transform.
Then the Yoneda algebra given by $\mathrm{Ext}_{\mathcal{D}^s(Bun_G)}(\mathcal{D}^s, \mathcal{D}^s)$,
Can stablish the endomorphism of critial level modules

Then is completed the sequence of critical Verma Modules (projective Harish-Chandra module to Whole sequence),
The global functor to the diagram in question until

$\Omega^\bullet[Op_{L_G}]$ is: ${}^L\Phi^\mu(\mathcal{M}) = \mathcal{M} \boxtimes \rho^\mu(\mathbb{V})$, [12] with ${}^L\Phi^\mu$, a Hecke functor

To their Spectrum (in the Hamiltonian variety)
We use the quantum cohomology space versión:

$$H^q(Bun_G \mathcal{D}^s) = \mathbb{H}^q_{G[[z]]}(\mathbf{G}, (\wedge^\bullet[\Sigma^0] \otimes \mathbb{V}_{critical}; \partial))$$

where $\mathbf{G} := G((z))/G[[\Sigma^0]]$, (the thick flag variety) where is clear that $\forall \phi \in G((z)), \phi \bullet G[[\Sigma^0]] \in \mathbf{G}$, then the elements of are the elements of $\mathfrak{g}[[z]]/G$, (where we are using directly the theorem 3. 1) which are in terms of graded vector space $SpecSymT$, the elements of $\mathrm{D}_{coh}({}^L Bun, \mathcal{D}^\times)$, which are included in the quasi-coherent category $M_{\mathcal{K}_F}(\hat{\mathfrak{g}}, Y)$. Finally $Spec_G^{\mathfrak{g}[[z]]/G}(\Omega^1(\mathbf{H})) = Y$.

Example As application to TFT, we consider the commutative diagram where a spectrum given by the theorem 3. 1, is the derived category $\mathcal{W}(H)$:

$$\begin{array}{ccccc} O_c(\varphi) \in H(mod\ f\ (C_{-*}(\Omega Z))) & \xrightarrow{R^{-1}} & H(\mathcal{M}) & \longrightarrow & C \\ \downarrow & & \downarrow embb & & \downarrow \\ C_{-*}(\Omega_x) & \xrightarrow{Diff} & \mathcal{W}(H) \ni \varphi & \xrightarrow{g} & M \end{array}$$

Thanks!!