Cosmology in Analytic Infinite Derivative (AID) gravity and its observational signatures

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Motivation

- **Gravity is not renormalizable**

- **Stelle's 1977 and 1978 papers show that R^2 gravity is renormalizable gravity with the price of a physical (Weyl) ghost**
  $(- + ++) \Rightarrow -\partial\varphi^2$ **is a good field**

- **Ostrogradski statement from 1850 forbids higher derivatives in general**

- **Starobinsky inflation is based on R^2 and works perfectly**

Exorcising ghosts

- **Constrained systems like GR**
- **Special theories like Horndeski models**
- **Infinitely man yderivatives**

String Field Theory in 5 min

- **Solid stuff**
- **Unitary**
- **Finite**
- **Describes all our fields universally**
- **From the embedding space-time point of view contains analytic infinite derivative operators**
- **Well, has its own unsolved puzzles**

Starting point

We start with

$$S = \int d^D x \sqrt{-g} \left(\mathcal{P}_0 + \sum_i \mathcal{P}_i \prod_I (\hat{\mathcal{O}}_{iI} \mathcal{Q}_{iI}) \right)$$

Here $\mathcal{P}$ and $\mathcal{Q}$ depend on curvatures and $\mathcal{O}$ are operators made of covariant derivatives.

Everywhere the respective dependence is *analytic*.

The most general action to consider

We are looking for the most general action which captures in full generality the properties of a linearized model around *maximally symmetric space-times (MSS)* given $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$.

The result is [arxiv.1602.08475]

$$S = \int d^D x \sqrt{-g} \Bigg(\frac{M_P^2 R}{2}$$
$$+ \frac{\lambda}{2} \left(R \mathcal{F}(\Box) R + L_{\mu\nu} \mathcal{F}_L(\Box) L^{\mu\nu} + W_{\mu\nu\lambda\sigma} \mathcal{F}_W(\Box) W^{\mu\nu\lambda\sigma} \right) - \Lambda \Bigg)$$

Here $L_{\mu\nu} = R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu}$ **and for any** X

$$\mathcal{F}_X(\Box) = \sum_{n \geq 0} f_{Xn} \Box^n$$

How come?

MSS

$$R_{\mu\nu\alpha\beta} = f(x)(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})$$

Bianchi identities dictate $f(x) = \frac{R_0}{(D-1)(D-2)}$

Consider term by term

$$R^2 \to h^2, \quad R^3 \to 3h^2 R, \ldots$$

$$\partial R^2 \to \partial h^2, \quad \partial R^2 R \to \partial h^2 R, \quad \partial R^3 \to 3\partial h^2 \partial R \to 0, \ldots$$

The same logic applies for $R_{\mu\nu}$ and even simpler for the Weyl tensor as it is zero on an MSS background.

Finally, We can fix any of f_0, f_{L0}, f_{W0} because the Gauss-Bonnet (GB) term is a topological invariant.

Even more!

Around MSS in $D = 4$ the derived action can be reduced further. Bianchi identities are so powerful that they allow to fix any of tree functions $\mathcal{F}$ and not only their constant Taylor coefficients. For example we can drop $\mathcal{F}_L$ entirely and remain with

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2 R}{2} + \frac{\lambda}{2}\left(R\mathcal{F}(\Box)R + W_{\mu\nu\lambda\sigma}\mathcal{F}_W(\Box)W^{\mu\nu\lambda\sigma}\right) - \Lambda\right)$$

Around MSS but in $D \geq 5$ the GB term is not a topological invariant and we are left with

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2 R}{2} + \frac{\lambda}{2}\left(R\mathcal{F}(\Box)R + f_{L0}L_{\mu\nu}^2 + W_{\mu\nu\lambda\sigma}\mathcal{F}_W(\Box)W^{\mu\nu\lambda\sigma}\right) - \Lambda\right)$$

Still, we are able to drop all higher derivative terms for $L_{\mu\nu}$

Quadratic action around (A)dS with $\bar{R} = 4\Lambda/M_P^2$

The covariant decomposition is

$$h_{\mu\nu} = \frac{2}{M_P^2} h^{\perp}_{\mu\nu} + \bar{\nabla}_\mu A_\nu + \bar{\nabla}_\nu A_\mu + (\bar{\nabla}_\mu \bar{\nabla}_\nu - \frac{1}{4}\frac{2}{M_P^2}\sqrt{\frac{8}{3}}\bar{g}_{\mu\nu}\bar{\Box})B + \frac{1}{4}\frac{2}{M_P^2}\sqrt{\frac{8}{3}}\bar{g}_{\mu\nu}h$$

Here $\bar{\nabla}^\mu h^{\perp}_{\mu\nu} = \bar{g}^{\mu\nu} h^{\perp}_{\mu\nu} = \bar{\nabla}^\mu A_\mu = 0$**.**

Spin-2:

$$S_2 = \frac{1}{2}\int dx^4 \sqrt{-\bar{g}}\ h^{\perp}_{\nu\mu}\left(\bar{\Box} - \frac{\bar{R}}{6}\right)\left[\mathcal{P}(\bar{\Box})\right] h^{\perp\mu\nu}$$

$$\mathcal{P}(\bar{\Box}) = 1 + \frac{2}{M_P^2}\lambda f_0 \bar{R} + \frac{\lambda}{M_P^2}\left\{\mathcal{F}_L(\bar{\Box})\left(\bar{\Box} - \frac{\bar{R}}{6}\right) + 2\mathcal{F}_W\left(\bar{\Box} + \frac{\bar{R}}{3}\right)\left(\bar{\Box} - \frac{\bar{R}}{3}\right)\right\}$$

Spin-0 (here $\phi \equiv \bar{\Box}B - h$**):**

$$S_0 = -\frac{1}{2}\int dx^4 \sqrt{-\bar{g}}\ \phi(3\bar{\Box} + \bar{R})\left[\mathcal{S}(\bar{\Box})\right]\phi$$

$$\mathcal{S}(\bar{\Box}) = 1 + \frac{2}{M_P^2}\lambda f_0 \bar{R} - \frac{\lambda}{M_P^2}\left\{2\mathcal{F}(\bar{\Box})(3\bar{\Box} + \bar{R}) + \frac{1}{2}\mathcal{F}_L\left(\bar{\Box} + \frac{2}{3}\bar{R}\right)\bar{\Box}\right\}$$

Physical excitations

Effectively we modify the propagators as follows

$$\Box - m^2 \to \mathcal{G}(\Box)$$

To preserve the physics we demand

$$\mathcal{G}(\Box) = (\Box - m^2)e^{\sigma(\Box)}$$

Here $\sigma(\Box)$ must be an *entire* function resulting that the exponent of it has no roots.

We arrange this in our model by virtue of functions $\mathcal{F}$. At this stage we can drop any one of three $\mathcal{F}$-s. The simplest choice is to drop $\mathcal{F}_L$.

Starobinsky inflation in non-local gravity

For any:

$$\Box R = r_1 R + r_2$$

We have a solution:

$$\mathcal{F}^{(1)}(r_1) = 0,\ \frac{r_2}{r_1}(\mathcal{F}(r_1) - f_0) = -\frac{M_P^2}{2\lambda} + 3r_1\mathcal{F}(r_1),\ 4\Lambda r_1 = -r_2 M_P^2$$

In the case of interest $\Lambda = 0$.

Notice that the we have started with the trace of Einstein equations in a local R^2 gravity.

Saying local gravity we do mean *any* including patological parameters in that local counterpart.

Choice of $\mathcal{F}(\Box)$

We should arrange that the theory is ghost-free meaning that no more than one pole arises in the scalar sector. The new degree of freedom is named scalaron and its mass is denoted as M. A possible form is:

$$\frac{\lambda}{M_P^2}\mathcal{F}(\Box) = -\frac{1}{6\Box}\left[e^{H_0(\Box)}\left(1-\frac{\Box}{M_P^2}\right)-1\right]$$

The conditions on $\mathcal{F}(\Box)$ imply that $H_0(\Box)$ is an entire function and moreover:

$$r_1 = M^2$$

$$H_0(r_1) = 0$$

Power spectra and r

Tensor modes

$$|\delta_h|^2 = \frac{H^2}{2\pi^2\lambda\mathcal{F}_1\bar{R}}e^{2\omega(\bar{R}/6)} \quad \textbf{where} \quad \mathcal{P}(\bar{\Box}) = e^{2\omega(\bar{\Box})}$$

Scalar modes (actually $\mathcal{R} = \Psi + \frac{H}{\dot{\bar{R}}}\delta R_{GI}$**)**

$$|\delta_{\mathcal{R}}|^2 \approx \frac{H^6}{16\pi^2\dot{H}^2}\frac{1}{3\lambda\mathcal{F}_1\bar{R}}$$

Tensor to scalar ratio r

$$r = \frac{2|\delta_h|^2}{|\delta_{\mathcal{R}}|^2} = 48\frac{\dot{H}^2}{H^4}e^{2\omega(\bar{R}/6)}$$

All quantities here are at the horizon crossing $k = Ha$**.**
Analogously

$$N = \int_{t_i}^{t_f} H dt = \frac{1}{2\epsilon_1} \;\Rightarrow\; r = 48\epsilon_1^2 e^{2\omega(\bar{R}/6)} = \frac{12}{N^2}e^{2\omega(\bar{R}/6)}$$

UV completeness

Minkowski propagator:

$$\Pi = -\left(\frac{P^{(2)}}{k^2 e^{H_2(-k^2)}} - \frac{P^{(0)}}{2k^2 e^{H_0(-k^2)}\left(1+\frac{k^2}{M^2}\right)}\right)$$

To guarantee that the QFT machinery works we arrange a polynomial decay of the propagator near infinity. The rate of the decay is our choice. Recall that we still need the functions $H_{0,2}$ to be entire.

An example of such a function can be, for instance

$$H \sim \Gamma\left(0, p(z)^2\right) + \gamma_E + \log\left(p(z)^2\right)$$

where $p(z)$ is a polynomial.

Beyond 1-loop the powercounting arguments work just like in the higher derivative regularization.

p-adic reformulation of the non-local gravity

The scalar part of the previous action is equivalent to the following one

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2 R}{2}\left(1+\frac{2}{M_P^2}\psi\right) - \frac{1}{2\lambda}\psi\frac{1}{\mathcal{F}(\Box)}\psi + \dots\right)$$

An important property here is the non-minimal coupling of a scalar field to gravity.

The conformal transform $\left(1+\frac{2}{M_P^2}\psi\right)^2 g_{\mu\nu} = \bar{g}_{\mu\nu}$ **allows us to decouple the gravity and the scalar field even more**

$$S = \int d^4x\sqrt{-\bar{g}}\left(\frac{M_P^2}{2}\bar{R} - \frac{M_P^2}{2}\frac{6}{(M_P^2+2\psi)^2}\bar{g}^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \frac{M_P^4}{2\lambda(M_P^2+2\psi)^2}\psi\mathcal{G}(\mathcal{D})\psi\right)$$

Here

$$\mathcal{G}(\mathcal{D}) = \frac{1}{\mathcal{F}(\mathcal{D})} \text{ and } \mathcal{D} = \left(1+\frac{2}{M_P^2}\psi\right)\Box_{\bar{g}} - \frac{2}{M_P^2}\bar{g}^{\mu\nu}\partial_\mu\psi\partial_\nu$$

Conclusions

- **A UV complete and unitary gravity is presented**
- **Starobinsky inflation is natively embedded in this model**
- **The theory predicts a modified value for r**
- **A connection to p-adic strings is outlined**
- **Few words about SFT in this story, Slavnov-Taylor identities, Cutkosky rules, etc.**

Open questions

- **Deeper study of the full Starobinsky model embedded in this non-local setup.**

- **Explicit computation of the one-loop divergences in this model**

- **This theory does not a priori prohibits a coexistence of a bounce and inflation. The question is to find such a configuration**

- **Derive the graviton action from the SFT in the full rigor.**

Thank you for listening!