New cosmological solutions in Nonlocal Modified Gravity

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Motivation

Large cosmological observational findings:

- High orbital speeds of galaxies in clusters. (F.Zwicky, 1933)
- High orbital speeds of stars in spiral galaxies. (Vera Rubin, at the end of 1960es)
- Accelerated expansion of the Universe. (1998)

Big Bang

- Another cosmological problem is related to the Big Bang singularity. Namely, under rather general conditions, general relativity yields cosmological solutions with zero size of the universe at its beginning, what means an infinite matter density.
- Note that when physical theory contains singularity, it is not valid in the vicinity of singularity and must be appropriately modified.

Problem solving approaches

There are two problem solving approaches:

- Dark matter and energy
- Modification of Einstein theory of gravity

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu},\ c = 1$$

where $T_{\mu\nu}$ is stress-energy tensor, $g_{\mu\nu}$ are the elements of the metric tensor, $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature of metric.

Dark matter and energy

- If Einstein theory of gravity can be applied to the whole Universe then the Universe contains about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.
- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.

Modification of Einstein theory of gravity

Motivation for modification of Einstein theory of gravity

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

Approaches to modification of Einstein theory of gravity

There are different approaches to modification of Einstein theory of gravity.

- Einstein General Theory of Relativity

From action $S = \int \left(\frac{R}{16\pi G} - L_m - 2\Lambda \right) \sqrt{-g} d^4x$ using variational methods we get field equations

Equations of Motion

$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}, \ c = 1$

where $T_{\mu\nu}$ is stress-energy tensor, $g_{\mu\nu}$ are the elements of the metric tensor, $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature of metric.

Modified Gravity: Kinds of modification

- First modifications: Einstein 1917, Weyl 1919, Edington 1923, ...

Einstein-Hilbert action

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} R + \int d^4x \sqrt{-g}\, \mathcal{L}(matter)$$

modification

$$R \rightarrow f(R, \Lambda, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, \dots), \quad \Box = \nabla^{\mu}\nabla_{\mu} = \frac{1}{\sqrt{-g}}\partial_{\mu}\sqrt{-g}g^{\mu\nu}\partial_{\nu}$$

Gauss-Bonnet invariant

$$\mathcal{G} = R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu}$$

Modified Gravity: Kinds of modification

- f(R) modified gravity

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} f(R) + \int d^4x \sqrt{-g}\, \mathcal{L}(matter)$$

- Gauss-Bonnet modified gravity

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} (R + \alpha \mathcal{G}) + \int d^4x \sqrt{-g}\, \mathcal{L}(matter)$$

- nonlocal modified gravity

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} f(R, \Box) + \int d^4x \sqrt{-g}\, \mathcal{L}(matter)$$

Nonlocal Modified Gravity

Under nonlocal modification of gravity we understand replacement of the scalar curvature R in the Einstein-Hilbert action by a suitable function $F(R, \Box)$, where $\Box = \nabla_\mu \nabla^\mu$ is dAlembert operator and ∇_μ denotes the covariant derivative.

Let M be a four-dimensional pseudo-Riemannian manifold with metric $(g_{\mu\nu})$ of signature $(1, 3)$. We consider a class of nonlocal gravity models without matter, given by the following action

$$S = \frac{1}{16\pi G} \int_M \left(R - 2\Lambda + \sqrt{R - 2\Lambda}\, \mathcal{F}(\Box) \sqrt{R - 2\Lambda} \right) \sqrt{-g}\, d^4x,$$

where $\mathcal{F}(\Box) = \sum_{n=1}^{\infty} f_n \Box^n$ and Λ is cosmological constant.

Nonlocal Modified Gravity

The previous action can be rewritten in the form

$$S = \frac{1}{16\pi G}\int_M \sqrt{R-2\Lambda}\, F(\Box)\, \sqrt{R-2\Lambda}\, \sqrt{-g}\; d^4x,$$

where $F(\Box) = 1 + \mathcal{F}(\Box) = 1 + \sum_{n=1}^{\infty} f_n \Box^n$.

FRW metric

We use Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right), \ k \in \{-1, 0, 1\}.$$

FRW metric

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$$R = \frac{6\left(a(t)\ddot{a}(t) + \dot{a}(t)^2 + k\right)}{a(t)^2}$$

In case of FRW metric the d'Alembert operator is

$$\Box R = -\ddot{R} - 3H\dot{R}, \qquad H = \frac{\dot{a}}{a}$$

Equations of Motion

By variation of action S with respect to metric $g^{\mu\nu}$ we obtain

$$
\begin{aligned}
& G_{\mu\nu} + \Lambda g_{\mu\nu} + (R_{\mu\nu} - \nabla_\mu \nabla_\nu + g_{\mu\nu}\Box) V^{-1}\mathcal{F}(\Box) V \\
& + \sum_{n=1}^{+\infty} \frac{f_n}{2} \sum_{l=0}^{n-1} \Big(g_{\mu\nu} (g^{\alpha\beta} \partial_\alpha \Box^l V \partial_\beta \Box^{n-1-l} V + \Box^l V \Box^{n-l} V) \\
& - 2\partial_\mu \Box^l V \partial_\nu \Box^{n-l-1} V \Big) - \frac{1}{2} g_{\mu\nu} V \mathcal{F}(\Box) V = 0,
\end{aligned}
$$

where $V = \sqrt{R - 2\Lambda}$.

The trace and 00-component of EOM

Suppose that manifold M has the FRW metric. Then we have two linearly independent equations (trace and 00-equation):

$$
\begin{aligned}
&4\Lambda - R - 2V\mathcal{F}(\Box)V + (R + 3\Box)V^{-1}\mathcal{F}(\Box)V \\
&+ \sum_{n=1}^{+\infty} f_n \sum_{l=0}^{n-1} (\partial_\mu \Box^l V \partial^\mu \Box^{n-1-l} V + 2\Box^l V \Box^{n-l} V) = 0, \\
&G_{00} + \Lambda g_{00} + (R_{00} - \nabla_0 \nabla_0 + g_{00}\Box)V^{-1}\mathcal{F}(\Box)V \\
&+ \sum_{n=1}^{+\infty} \frac{f_n}{2} \sum_{l=0}^{n-1} \Big(g_{00}(g^{\alpha\beta}\partial_\alpha \Box^l V \partial_\beta \Box^{n-1-l} V + \Box^l V \Box^{n-l} V) \\
&- 2\partial_0 \Box^l V \partial_0 \Box^{n-l-1} V \Big) - \frac{1}{2} g_{00} V \mathcal{F}(\Box) V = 0,
\end{aligned}
$$

where $R_{00} = -3\frac{\ddot{a}}{a}, \quad G_{00} = 3\frac{\dot{a}^2 + k}{a^2}.$

Ansatz

In order to solve equations of motion we use the following ansatz:

$$\Box\sqrt{R-2\Lambda} = p\,\sqrt{R-2\Lambda},$$

where p is a constant.

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where p is a constant.

The first consequences of ansatz are:

$$\Box^n\sqrt{R-2\Lambda} = p^n\sqrt{R-2\Lambda},\ \ n \geq 0$$
$$\mathcal{F}(\Box)\sqrt{R-2\Lambda} = \mathcal{F}(p)\sqrt{R-2\Lambda}.$$

Cosmological solution $a(t) = Ae^{\gamma t^2}$, $\Lambda = 6\gamma$, $\gamma \neq 0$, $k = 0$.

We consider scale factor of the form

$$a(t) = Ae^{\gamma t^2}.$$

The following anzats

$$\Box\sqrt{R - 12\gamma} = -6\gamma\sqrt{R - 12\gamma}$$

is satisfied.

Cosmological solution $a(t) = Ae^{\gamma t^2}$, $\Lambda = 6\gamma$, $\gamma \neq 0$, $k = 0$.

We consider scale factor of the form

$$a(t) = Ae^{\gamma t^2}.$$

The following anzats

$$\Box\sqrt{R - 12\gamma} = -6\gamma\sqrt{R - 12\gamma}$$

is satisfied. Direct calculation shows that

$$R(t) = 12\gamma(1 + 4\gamma t^2),$$
$$\dot{R} = 96\gamma^2 t,$$
$$\Box\sqrt{R - 12\gamma} = -24\sqrt{3}\gamma|\gamma||t|,$$
$$\Box^n\sqrt{R - 12\gamma} = (-6\gamma)^n\sqrt{R - 12\gamma}, \ n \geq 0,$$
$$\mathcal{F}(\Box)\sqrt{R - 12\gamma} = \mathcal{F}(-6\gamma)\sqrt{R - 12\gamma}.$$

Cosmological solution $a(t) = Ae^{\gamma t^2}$, $\Lambda = 6\gamma$, $\gamma \neq 0$, $k = 0$.

Substituting $a(t)$ into trace equation we get the following system of equations:

$$\Lambda - 3\gamma + 3\gamma\mathcal{F}(-6\gamma) - 12\gamma^2\mathcal{F}'(-6\gamma) = 0,$$
$$-\gamma - \gamma\mathcal{F}(-6\gamma) - 12\gamma^2\mathcal{F}'(-6\gamma) = 0.$$

In order to satisfy the last system of equations we have:

$$\mathcal{F}(-6\gamma) = -1,$$
$$\mathcal{F}'(-6\gamma) = 0.$$

Cosmological solution $a(t) = Ae^{\gamma t^2}$, $\Lambda = 6\gamma$, $\gamma \neq 0$, $k = 0$.

We have:

$$
\begin{aligned}
R_{00} &= -12\gamma^2 t^2 - 6\gamma, \\
H(t) &= 2\gamma t, \\
G_{00} &= 12\gamma^2 t^2.
\end{aligned}
$$

When we substitute these conditions into 00 equation we obtain

$$\Lambda = 6\gamma.$$

Cosmological solution $a(t) = Ae^{\gamma t^2}$, $\Lambda = 6\gamma$, $\gamma \neq 0$, $k = 0$.

We conclude that the equations of motion are satisfied if and only if

$$
\begin{aligned}
&\Lambda = 6\gamma, \ \gamma \neq 0, \\
&\mathcal{F}(-6\gamma) = -1, \\
&\mathcal{F}'(-6\gamma) = 0.
\end{aligned}
$$

Cosmological solution $a(t) = A\ t^{2/3} e^{\gamma t^2}$, $\Lambda = 14\gamma$, $\gamma \neq 0$, $k = 0$.

We consider scale factor of the form

$$a(t) = A\ t^{2/3} e^{\gamma t^2}.$$

The following anzats

$$\Box\sqrt{R - 28\gamma} = -6\gamma\sqrt{R - 28\gamma}$$

is satisfied.

Cosmological solution $a(t) = A\, t^{2/3} e^{\gamma t^2}$, $\Lambda = 14\gamma$, $\gamma \neq 0$, $k = 0$.

We consider scale factor of the form

$$a(t) = A\, t^{2/3} e^{\gamma t^2}.$$

The following anzats

$$\Box \sqrt{R - 28\gamma} = -6\gamma \sqrt{R - 28\gamma}$$

is satisfied. Direct calculation shows that

$$R(t) = 44\gamma + \frac{4}{3} t^{-2} + 48\gamma^2 t^2,$$

$$\dot{R} = 96\gamma^2 t - \frac{8}{3} t^{-3},$$

$$\Box^n \sqrt{R - 28\gamma} = (-6\gamma)^n \sqrt{R - 28\gamma}, \; n \geq 0,$$

$$\mathcal{F}(\Box) \sqrt{R - 28\gamma} = \mathcal{F}(-6\gamma) \sqrt{R - 28\gamma}.$$

Cosmological solution $a(t) = A\, t^{2/3} e^{\gamma t^2}$, $\Lambda = 14\gamma$, $\gamma \neq 0$, $k = 0$.

Substituting $a(t)$ into trace equation we get the following system of equations:

$$
\begin{aligned}
\mathcal{F}'(-6\gamma) &= 0, \\
\Lambda - 11\gamma + 3\gamma\mathcal{F}(-6\gamma) &= 0, \\
\mathcal{F}(-6\gamma) &= -1, \\
-\gamma^2 - \gamma^2\mathcal{F}(-6\gamma) &= 0.
\end{aligned}
$$

Cosmological solution $a(t) = A\, t^{2/3} e^{\gamma t^2}$, $\Lambda = 14\gamma$, $\gamma \neq 0$, $k = 0$.

We have:

$$\begin{aligned} R_{00} &= \frac{2}{3} t^{-2} - 12\gamma^2 t^2 - 14\gamma, \\ H(t) &= \frac{2}{3} t^{-1} + 2\gamma t, \\ G_{00} &= \frac{4}{3} t^{-2} + 12\gamma^2 t^2 + 8\gamma. \end{aligned}$$

Substituting this into the 00 component of EOM we obtain the following system of equations:

$$\begin{aligned} \mathcal{F}'(-6\gamma) &= 0, \\ 8\gamma - \Lambda - 6\gamma \mathcal{F}(-6\gamma) &= 0, \\ \mathcal{F}(-6\gamma) &= -1, \\ \gamma^2 + \gamma^2 \mathcal{F}(-6\gamma) &= 0. \end{aligned}$$

Cosmological solution $a(t) = A\, t^{2/3} e^{\gamma t^2}$, $\Lambda = 14\gamma$, $\gamma \neq 0$, $k = 0$.

The last two systems of equations are satisfied if and only if

$$\mathcal{F}(-6\gamma) = -1,$$
$$\mathcal{F}'(-6\gamma) = 0,$$
$$\Lambda = 14\gamma, \ \gamma \neq 0.$$

Cosmological solution $a(t) = Ae^{\lambda t}$, $\Lambda = 6\lambda^2$, $\lambda \neq 0$, $k = \pm 1$

We consider scale factor of the form

$$a(t) = A\, e^{\lambda t}.$$

The following anzats

$$\Box\sqrt{R - 2\Lambda} = 2\lambda^2\sqrt{R - 2\Lambda}$$

is satisfied.

Cosmological solution $a(t) = Ae^{\lambda t}$, $\Lambda = 6\lambda^2$, $\lambda \neq 0$, $k = \pm 1$

We consider scale factor of the form

$$a(t) = A\, e^{\lambda t}.$$

The following anzats

$$\Box\sqrt{R - 2\Lambda} = 2\lambda^2\sqrt{R - 2\Lambda}$$

is satisfied. Direct calculation shows that

$$R(t) = \frac{6k}{A^2} e^{-2\lambda t} + 12\lambda^2,$$

$$\Box^n\sqrt{R - 2\Lambda} = (2\lambda^2)^n\sqrt{R - 2\Lambda}, \; n \geq 0,$$

$$\mathcal{F}(\Box)\sqrt{R - 2\Lambda} = \mathcal{F}(2\lambda^2)\sqrt{R - 2\Lambda}.$$

Cosmological solution $a(t) = Ae^{\lambda t}$, $\Lambda = 6\lambda^2$, $\lambda \neq 0$, $k = \pm 1$

The trace and 00 equations become

$$4\Lambda - R + (4\Lambda - R)\mathcal{F}(2\lambda^2) - \frac{\dot{R}^2}{4(R-2\Lambda)}\mathcal{F}'(2\lambda^2) + 4(R-2\Lambda)\lambda^2\mathcal{F}'(2\lambda^2) = 0,$$

$$G_{00} - \Lambda + R_{00}\mathcal{F}(2\lambda^2) + \frac{1}{2}(R-2\Lambda)\mathcal{F}(2\lambda^2) - \frac{\dot{R}^2}{8(R-2\Lambda)}\mathcal{F}'(2\lambda^2) - \lambda^2(R-2\Lambda)\mathcal{F}'(2\lambda^2) = 0.$$

Cosmological solution $a(t) = Ae^{\lambda t}$, $\Lambda = 6\lambda^2$, $\lambda \neq 0$, $k = \pm 1$

The last two equations are satisfied if and only if

$$\Lambda = 6\lambda^2, \ \lambda \neq 0$$
$$\mathcal{F}(2\lambda^2) = -1,$$
$$\mathcal{F}'(2\lambda^2) = 0.$$

Cosmological solutions with constant scalar curvature

We want to find solution of equations of motion for cosmological scale factor $a(t)$ when $R = R_0 = constant$. It is useful to start from the differential equation

$$6\Big(\frac{\ddot{a}}{a} + \big(\frac{\dot{a}}{a}\big)^2 + \frac{k}{a^2}\Big) = R_0.$$

The change of variable $b(t) = a^2(t)$ yields second order linear differential equation with constant coefficients

$$3\ddot{b} - R_0 b + 6k = 0.$$

Cosmological solutions with constant scalar curvature

Depending on the sign of R_0 we have the following general solutions for $b(t)$:

$$R_0 > 0, \qquad b(t) = \frac{6k}{R_0} + \sigma \cosh \sqrt{\frac{R_0}{3}} t + \tau \sinh \sqrt{\frac{R_0}{3}} t,$$

$$R_0 = 0, \qquad b(t) = -kt^2 + \sigma t + \tau,$$

$$R_0 < 0, \qquad b(t) = \frac{6k}{R_0} + \sigma \cos \sqrt{\frac{-R_0}{3}} t + \tau \sin \sqrt{\frac{-R_0}{3}} t,$$

where σ and τ are some constants.

Cosmological solutions with constant scalar curvature

When we substitute $R = R_0 = constant \neq 2\Lambda$ into the equations of motion we get condition

$$R_0 + 4R_{00} = 0.$$

Equations of motion are satisfied without conditions on function $\mathcal{F}(\Box)$, because $\Box\sqrt{R - 2\Lambda} = 0$.
Consider now constraints which equation $R_0 + 4R_{00} = 0$ implies on the parameters σ, τ, k and R_0. Since $R_{00} = -3\frac{\ddot{a}}{a} = \frac{3}{4}\frac{(\dot{b})^2 - 2b\ddot{b}}{b^2}$, it follows the following connections between parameters:

$$\begin{aligned} R_0 > 0, &\qquad 36k^2 = R_0^2(\sigma^2 - \tau^2), \\ R_0 = 0, &\qquad \sigma^2 + 4k\tau = 0, \\ R_0 < 0, &\qquad 36k^2 = R_0^2(\sigma^2 + \tau^2). \end{aligned}$$

Cosmological solutions with $R_0 > 0$.

In this case, it is convenient to take $R_0 = 4\Lambda > 0$. Hence, scale factor $a(t)$ is

$$a(t) = \sqrt{\frac{3k}{2\Lambda} + \sigma \cosh \sqrt{\frac{4\Lambda}{3}} t + \tau \sinh \sqrt{\frac{4\Lambda}{3}} t}.$$

Moreover, let $\sigma^2 - \tau^2 > 0$, then we choose φ such that $\cosh \varphi = \frac{\sigma}{\sqrt{\sigma^2 - \tau^2}}$ and $\sinh \varphi = \frac{\tau}{\sqrt{\sigma^2 - \tau^2}}$, and we can write $a(t)$ as

$$a(t) = \sqrt{\frac{3k}{2\Lambda} + \sqrt{\sigma^2 - \tau^2} \cosh \left(\sqrt{\frac{4\Lambda}{3}} t + \varphi \right)}, \quad k = \pm 1.$$

Cosmological solutions with $R_0 > 0$.

Since $\frac{3}{2\Lambda} = \sqrt{\sigma^2 - \tau^2}$, one can rewrite $a(t)$ in the form

$$a(t) = \sqrt{\frac{3\left(\cosh\left(\sqrt{\frac{4\Lambda}{3}}t + \varphi\right) + k\right)}{2\Lambda}}, \quad k = \pm 1.$$

Now, let $\sigma^2 - \tau^2 = 0$, then the scale factor takes the form

$$a(t) = \sqrt{\sigma} e^{\pm\sqrt{\frac{\Lambda}{3}}t}, \quad k = 0, \, \sigma > 0.$$

Cosmological solutions with $R_0 = 4\Lambda > 0$.

There are three cases:

- Case $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{3}}t},\ k = 0$.
  One has $H(t) = \pm\sqrt{\frac{\Lambda}{3}},\ G_{00} = \Lambda$.
- Case $a(t) = \sqrt{\frac{3}{\Lambda}} \cosh \sqrt{\frac{\Lambda}{3}} t,\ k = +1$.
  Now $H(t) = \sqrt{\frac{\Lambda}{3}} \tanh \sqrt{\frac{\Lambda}{3}} t,\ G_{00} = \Lambda$.
- Case $a(t) = \sqrt{\frac{3}{\Lambda}} \mid \sinh \sqrt{\frac{\Lambda}{3}} t \mid,\ k = -1$.
  Here $H(t) = \sqrt{\frac{\Lambda}{3}} \coth \sqrt{\frac{\Lambda}{3}} t,\ G_{00} = \Lambda$.

Cosmological solutions with $R_0 < 0$.

In the third case, $R_0 < 0$ it is convenient to take $R_0 = -4 \mid \Lambda \mid$. Hence, scale factor $a(t)$ is

$$a(t) = \sqrt{-\frac{3k}{2 \mid \Lambda \mid} + \sigma \cos \sqrt{\frac{4 \mid \Lambda \mid}{3}} t + \tau \sin \sqrt{\frac{4 \mid \Lambda \mid}{3}} t}.$$

Moreover, if we choose φ such that $\cos \varphi = \frac{\sigma}{\sqrt{\sigma^2+\tau^2}}$ and $\sin \varphi = \frac{\tau}{\sqrt{\sigma^2+\tau^2}}$ we can rewrite it as

$$a(t) = \sqrt{-\frac{3k}{2 \mid \Lambda \mid} + \sqrt{\sigma^2 + \tau^2} \cos(\sqrt{\frac{4 \mid \Lambda \mid}{3}} t - \varphi)}, \quad k = -1.$$

Cosmological solutions with $R_0 < 0$.

Since in this case $\frac{3}{2|\Lambda|} = \sqrt{\sigma^2 + \tau^2}$, solution $a(t)$ can be presented in the form

$$a(t) = \sqrt{\frac{3}{2 \mid \Lambda \mid}\Big(\cos(\sqrt{\frac{4 \mid \Lambda \mid}{3}}t - \varphi) + 1\Big)}, \quad (k = -1).$$

Cosmological solution with $R_0 = -4 \mid \Lambda \mid < 0$.

The corresponding solution has the form

$$a(t) = \sqrt{-\frac{3}{\Lambda}} \mid \cos \sqrt{-\frac{\Lambda}{3}} t \mid,$$

where Λ is negative cosmological constant. In this case

$$H(t) = -\sqrt{-\frac{\Lambda}{3}} \tan \sqrt{-\frac{\Lambda}{3}} t,$$
$$G_{00} = - \mid \Lambda \mid, \; k = -1.$$

Conclusion

- We have considered a class of nonlocal gravity models with cosmological constant Λ and without matter, given by

$$S = \frac{1}{16\pi G} \int \left(R - 2\Lambda + \sqrt{R - 2\Lambda}\,\mathcal{F}(\Box)\,\sqrt{R - 2\Lambda}\,\right) \sqrt{-g}\; d^4x.$$

- Using ansatz $\Box\sqrt{R - 2\Lambda} = p\sqrt{R - 2\Lambda}$ we found some solutions:
- The solution $a(t) = A\, e^{\frac{\Lambda}{6} t^2}, \quad \Lambda \neq 0,\ k = 0.$
- The solution $a(t) = A\, t^{2/3} e^{\frac{\Lambda}{14} t^2}, \quad \Lambda \neq 0,\ k = 0.$
- The solution $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{6}}\, t},\ \Lambda > 0,\ k = \pm 1.$

Conclusion

Cosmological solutions with constant $R(t) = 4\Lambda > 0$

- $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{3}}t}, \ k = 0.$
- $a(t) = \sqrt{\frac{3}{\Lambda}} \cosh \sqrt{\frac{\Lambda}{3}}t, \ k = +1.$
- $a(t) = \sqrt{\frac{3}{\Lambda}} \mid \sinh \sqrt{\frac{\Lambda}{3}}t \mid, \ k = -1.$

Cosmological solutions with constant $R(t) = -4 \mid \Lambda \mid < 0$

- $a(t) = \sqrt{-\frac{3}{\Lambda}} \mid \cos \sqrt{-\frac{\Lambda}{3}}t \mid, \ \Lambda < 0, \ k = -1.$

References

- Biswas, T., Mazumdar, A., Siegel, W: Bouncing universes in string-inspired gravity. J. Cosmology Astropart. Phys. 0603, 009 (2006) [arXiv:hep-th/0508194]
- Biswas, T., Koivisto, T., Mazumdar, A.: Towards a resolution of the cosmological singularity in non-local higher derivative theories of gravity. J. Cosmology Astropart. Phys. 1011, 008 (2010) [arXiv:1005.0590v2 [hep-th]]
- Koshelev, A.S., Vernov, S.Yu.: On bouncing solutions in non-local gravity. [arXiv:1202.1289v1 [hep-th]]
- Dimitrijevic, I., Dragovich, B., Grujic J., Rakic, Z.: New cosmological solutions in nonlocal modified gravity. Rom. Journ. Phys. 58 (5-6), 550559 (2013) [arXiv:1302.2794 [gr-qc]]
- T. Biswas, A. S. Koshelev,A. Mazumdar, S. Yu. Vernov, "Stable bounce and inflation in non-local higher derivative cosmology", JCAP **08** (2012) 024, [arXiv:1206.6374v2 [astro-ph.CO]].
- I. Dimitrijevic, B.Dragovich, J. Grujic, Z. Rakic, "A new model of nonlocal modified gravity", Publications de l'Institut Mathematique **94** (108), 187–196 (2013).

References

I. Dimitrijevic, B. Dragovich, J. Grujic, A. S. Koshelev, and Z. Rakic, Cosmology of modified gravity with a non-local f(R). 2015, [arXiv:1509.04254].

I. Dimitrijevic, B.Dragovich, J. Stankovic, A. S. Koshelev, Z. Rakic, "On Nonlocal Modified Gravity and its Cosmological Solutions", Springer Proceedings in Mathematics and Statistics (2016).

I. Dimitrijevic, B. Dragovich, Z. Rakic and J.Stankovic, On Nonlocal Gravity with Constant Scalar Curvature, Publications de l'Institut Mathematique (2017) in review

I. Dimitrijevic, B. Dragovich, A.S. Koshelev, Z. Rakic and J.Stankovic, Cosmological solutions of a new nonlocal modified gravity, (2017), in preparation