ADS 4D/BPS 3D Correspondence

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Outline

A Brief History of Monopoles

SUSY: 4D $\rightarrow$ 3D $\times S^1$

N=2 SUSY in 4D

Standard Model

Conclusions
Figure 1. Static configuration of an electric change and a magnetic monopole.

which follows from symmetry (the integral can only supply a numerical factor, which turns out to be $4\pi^2$). The quantization of charge follows by applying semiclassical quantization of angular momentum:

$$J \cdot \hat{R} = eg c = n\hbar$$

$n = 0, \pm 1, \pm 2, \ldots$ (2.4a)

or

$$eg = m'\hbar c, m' = n^2.$$ (2.4b)

(Here, and in the following, we use $m'$ to designate this “magnetic quantum number.” The prime will serve to distinguish this quantity from an orbital angular momentum quantum number, or even from a particle mass.)

2.3. Classical scattering

Actually, earlier in 1896, Poincaré [3] investigated the motion of an electron in the presence of a magnetic pole. This was inspired by a slightly earlier report of anomalous motion of cathode rays in the presence of a magnetized needle [32]. Let us generalize the analysis to two dyons (a term coined by Schwinger in 1969 [11]) with charges $e_1, g_1, e_2, g_2$, respectively. There are two charge combinations $q = e_1 e_2 + g_1 g_2, \kappa = -e_1 g_2 - e_2 g_1 c$. (2.5)

Then the classical equation of relative motion is ($\mu$ is the reduced mass and $v$ is the relative velocity)

$$\mu \frac{d^2}{dt^2} \mathbf{r} = q \mathbf{r} \mathbf{r}^{-3} - \kappa \mathbf{v} \times \mathbf{r} \mathbf{r}^{-3}.$$ (2.6)

The constants of the motion are the energy and the angular momentum,

$$E = \frac{1}{2} \mu v^2 + q \mathbf{r}, J = \mathbf{r} \times \mu \mathbf{v} + \kappa \hat{r}.$$ (2.7)

Note that Thomson’s angular momentum (2.3) is prefigured here. Because $J \cdot \hat{r} = \kappa$, the motion is confined to a cone, as shown in figure 2. Here the angle of the cone is given by

$$\cot \chi = \frac{l}{|\kappa|}, l = \mu v_0 b,$$ (2.8)

where $v_0$ is the relative speed at infinity, and $b$ is the impact parameter. The scattering angle $\theta$ is given by

$$\cos \frac{\theta}{2} = \cos \chi \frac{|\kappa|}{|\kappa|} \sin \left(\frac{\xi}{2} \cos \frac{\chi}{2}\right).$$ (2.9a)
Dirac

charge quantization

$qg = \frac{n}{2}$

Proc. Roy. Soc. Lond. A133 (1931) 60
\'t Hooft-Polyakov

topological monopoles

hedgehog gauge

\[ \phi^a = \hat{r} v h(\nu r) \]

\[ W_i^a = \epsilon^{air} \hat{r}^j \frac{f(\nu r)}{gr} \]
\[ \phi^a = \hat{r} v h(vr) \]

\[ W_i^a = \epsilon^{air} \hat{r}^j f(vr) \frac{f(vr)}{gr} \]

\[ U^\dagger \tau^a \phi^a U = v h(vr) \tau^3 \]

\[ U = \frac{1}{\sqrt{2}} \left( \sqrt{1 + \hat{r}_3} I + i \frac{\hat{r}_2 \sigma^1 - \hat{r}_1 \sigma^2}{\sqrt{1 + \hat{r}_3}} \right) \]
`t Hooft-Mandelstam

magnetic condensate confines electric charge

Phys. Rept. 23 (1976) 245
4D \rightarrow \text{3D} \times S^1

\text{SUSY SU(N) with F flavors}

W_\mu^a \rightarrow \tilde{W}, \phi^a

\text{monopole solution}
$$4D \rightarrow 3D \times S^1$$

Wick rotation

monopole solution
4D $\rightarrow$ 3D $\times S^1$

compactify

monopole solution
N-1 Embeddings of SU(2)

N-1 diagonal generators

\[
\begin{pmatrix}
\frac{1}{2} & 0 & 0 & \cdots \\
0 & -\frac{1}{2} & 0 & \cdots \\
0 & 0 & 0 & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\begin{pmatrix}
0 & 0 & 0 & \cdots \\
0 & \frac{1}{2} & 0 & \cdots \\
0 & 0 & -\frac{1}{2} & \cdots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\begin{pmatrix}
0 & 0 & 0 & \cdots \\
0 & 0 & 0 & \frac{1}{2} & \cdots \\
0 & 0 & 0 & \frac{1}{2} & \cdots \\
\vdots & \vdots & \vdots & \ddots & \ddots
\end{pmatrix}
\ldots
\]

monopole solutions
Roots of SU(3)

\[
\mathbf{H} = (T^3, T^8)
\]

\[
\begin{pmatrix}
\frac{1}{2} & 0 & 0 \\
0 & -\frac{1}{2} & 0 \\
0 & 0 & 0
\end{pmatrix}
= \alpha \cdot \mathbf{H}
\]

\[
\begin{pmatrix}
0 & 0 & 0 \\
0 & \frac{1}{2} & 0 \\
\frac{1}{2} & 0 & -\frac{1}{2}
\end{pmatrix}
= \beta \cdot \mathbf{H}
\]

\[
\alpha = (1, 0) \quad \beta = (-\frac{1}{2}, \frac{\sqrt{3}}{2})
\]
N-1 Embeddings of SU(2)

N-1 diagonal generators

\[ \alpha_1 \cdot H \]
\[ \alpha_2 \cdot H \]
\[ \alpha_3 \cdot H \]
\[ \ldots \]

monopole charges

\[ \alpha_1 \]
\[ \alpha_2 \]
\[ \alpha_3 \]
\[ \ldots \]
Roots of SU(3)

\[ H = (T^3, T^8) \]

\[ \langle \phi \rangle = a \cdot H \]

\[ a = v_1 \alpha_1 + v_2 \beta \]

\[ \alpha = (1, 0) \quad \beta = \left( -\frac{1}{2}, \frac{\sqrt{3}}{2} \right) \]
Roots of SU(3)

\[ H = (T^3, T^8) \]

\[ \langle \phi \rangle = a \cdot H \]

\[ a = \nu_1 \alpha_1 + \nu_2 \beta \]

\[ \alpha = (1, 0) \quad \beta = \left( -\frac{1}{2}, \frac{\sqrt{3}}{2} \right) \]
Monopole Solutions

\[ \langle \phi \rangle = a \cdot H \]
\[ a = v_1 \alpha + v_2 \beta \]

\[ \phi = v_1 \alpha \cdot H + \hat{r}^a T^a_\beta v_1 h(v_2 r) ; \quad T^3_\beta = \beta \cdot H \]

\[ \phi = v_2 \beta \cdot H + \hat{r}^a T^a_\alpha v_2 h(v_1 r) ; \quad T^3_\alpha = \alpha \cdot H \]
4D \rightarrow 3D \times S^1

Wick rotation

monopole solution
$4D \rightarrow 3D \times S^1$

KK monopole solution
\[ 3D \times S^1 \rightarrow 4D \]

N-1 monopole solutions + KK monopole

\[ \rightarrow 4D \text{ instanton as } R \rightarrow \infty \]
Instanton Zero Modes

2N gauginos

2F quarks
Instanton Zero Modes

2N gauginos

Poppitz & Unsal hep-th/0812.2085
Instanton Zero Modes

\[ F = N - 1 \]

\[ 2N - 2 \]

\[ \text{fermion mass} = \frac{\partial W}{\partial Q \partial \bar{Q}} \]
Instanton Superpotential

\[ W = \frac{\Lambda^{3N-F} \det Q^\dagger \overline{Q}^\dagger}{|\det \overline{Q}\overline{Q}|^2} = \frac{\Lambda^{3N-F}}{\det \overline{Q}\overline{Q}} \]
$F < N$

$W_{\text{ADS}} = (N - F) \left( \frac{\Lambda^{3N-F}}{\det Q \overline{Q}} \right)^{\frac{1}{N-F}}$

where does this come from?
$W_{3D} = \sum_i \frac{1}{Y_i}$

$Y_i = e^{\mathbf{a} \cdot \alpha_i + i \gamma_i}$

$\phi = \mathbf{a} \cdot \mathbf{H}$

$\partial_m \gamma_i = \epsilon_{mnp} F_{i}^{np}$

$R \to 0$

Finite $R$

\[ W = \sum_{i} \frac{1}{Y_i} + \eta Y_{KK} \]
Mixed Coulomb Branch
SU(3) with F=1

\[ \phi = \frac{1}{2} \text{diag}(v, 0, -v) \]

SU(3)→U(1)×U(1)

\[ Q = \overline{Q} = \begin{pmatrix} 0 \\ q \\ 0 \end{pmatrix} \]

SU(3)→SU(2)

SU(3)→U(1)

monopoles are confined
Mixed Coulomb Branch
SU(3) with F=1

monopoles are confined
superHiggs mechanism gives fermions masses
Mixed Coulomb Branch
SU(3) with F=1

\[ W = \eta Y_1 Y_2 + \frac{1}{Y_1 Y_2 Q\bar{Q}} \]

\[ W = 2 \left( \frac{\eta}{\text{det } QQ} \right)^{\frac{1}{2}} \]
Mixed Coulomb Branch
SU(3) with F=1

$q \gg \frac{1}{R}, v$

SU(3)→SU(2) in "4D", F=0

$\Lambda^8 = \Lambda^6_L q^2$

$W = \eta_L Y_L + \frac{1}{Y_L}$

$\phi = a \cdot H$

$a = v(\alpha + \beta)$

matches, since

$Y_L \propto Y_1 Y_2 q^2$

$\eta_L = \frac{\eta}{q^2}$
SU(N) with $F < N-1$

- $\phi$ has $F$ zeros
- $Q, \overline{Q}$ have $F$ VEVs
- $SU(N) \rightarrow SU(F) \times U(1)^{N-F}$
- $SU(N) \rightarrow SU(N-F)$
- $SU(N) \rightarrow U(1)^{N-F-1}$

- $F+1$ monopoles are confined
- $2F$ gauginos get masses
- $2(F+1) - 2F = 2$
- $2$ gaugino legs $\Rightarrow$ ADS super potential
Conclusions

Monopoles are still fascinating after all these years

Confined monopoles relate 3D BPS monopoles to the 4D ADS superpotential